TEMPERATURE SENSORS

subjects:

Thermistors and RTD
Diodes and PTAT integrated sensors
Thermocouples

Temperature

- Temperature controls the direction and the amount of the heat exchanged by systems “in thermal contact”
- Given an ensemble of particles, their temperature is connected to the average kinetic energy
  - Equipartition theorem
    - N: degree of freedom

$$T = \frac{1}{2} m \cdot \langle v^2 \rangle = \frac{N}{2} k \cdot T$$

- Unit of measure: Kelvin (K)
  - Practical unit: centigrade scale (Celsius) 0°C=273.15 K
    - Temperature differences (ΔT) measured in Kelvin and in Celsius are identical.
  - Fahrenheit scale is the most known alternative:

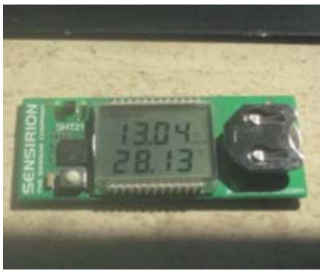
$$T[°F] = 32 + 1.8 \cdot T[°C]$$

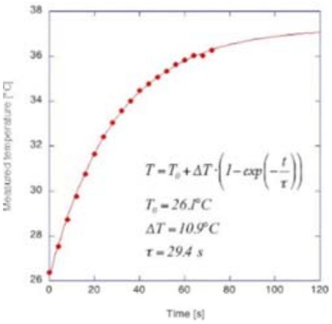
- All natural phenomena depends on temperature; then, in general transducers of temperature may be based on physical, chemical and even biological principles.
- For sensors, it is convenient to exploit the phenomena involving a relationship between electric conductivity and temperature.
  - Resistors (metals and semiconductors)
  - Junction:
    - semiconductor – semiconductor (diodes and integrated circuits)
    - conductor – conductor (thermocouples)

Temperature sensors

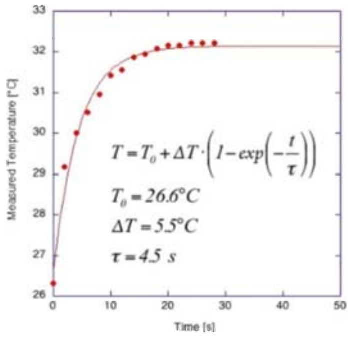
- Two main families: in-contact (conduction + convection) and non-contact (radiation) sensors
  - non-contact sensors: IR detectors
- In-contact sensors are sensitive to their own temperature, thus to be meaningful it has to be identical to the temperature of the object under measurement.
  - Thermal contact → heat exchange: conduction, convection, radiation.
- The sensor should not perturb the temperature of the body under measurement
  - Small mass and thermal capacity
- The response time depends on the time necessary to equilibrate the temperature of the sensor with the temperature under measurement.

Radiation+convection





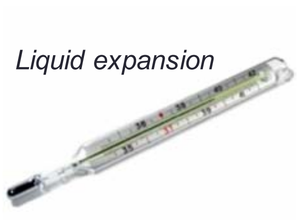
contact



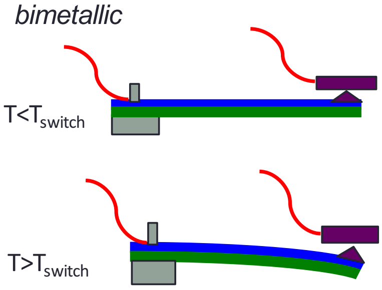
Expansion thermometer

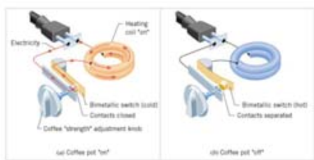
- The forces keeping together atoms and molecules in solids and liquids are sensitive to the temperature. Then, as the temperature changes, the equilibrium position of atoms changes, and the geometrical dimensions change.
- The measurement of the variation of the space occupied by bodies provides a method for temperature measurement.
- Due possibilities:
  - Liquid expansion
    - Example: mercury thermometer
      - Expansion is forced along a capillary tube, then the volume changes are measured as length changes.
  - Solid expansion
    - e.g. bimetallic slabs
      - Two materials of different expansion coefficent are solded together, the different expansion forces the slab to bend
      - Slabs can be shaped as a spring, then the expansion results in a rotation.

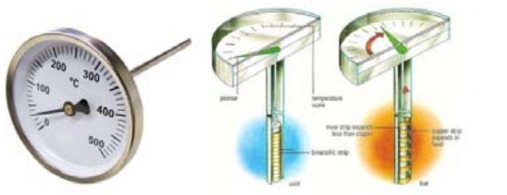
Liquid expansion



bimetallic







Temperature sensors based on change of conductivity

- Electric conductivity intrinsically depends on temperature.
- The resistive temperature sensors are called *Thermistor.*
- Thermistors are classified respect to the sign of the sensitivity (dR/dT).
  - PTC (positive temperature coefficient)
  - NTC (negative temperature coefficient)
- Semiconductor thermistors can be either PTC or NTC (depends on doping). Metallic thermistors are always PTC.
- The name thermistor is used for semiconductors, while metallic sensors are called *Resistance Temperature Detectors* (RTD).

Thermal effects on mobility and charge carriers density

- The conductivity of a generic material (σ=J/E) is:

$$\sigma = q \cdot \left(n \cdot \mu_n + p \cdot \mu_p \right)$$

  - q: elementary charge; n and p: density of electrons and holes; μ_n and μ_p mobility of electrons and holes.
- An increase of temperature results in an increased thermal motion of the atoms, then the scattering probability of electrons and holes increases. *As a consequence, the mobility always decreases with the temperature.*
- **Metals**:
  - Only electrons. Their density does not depend on temperature. Thus, the temperature affects only the mobility. Therefore, the conductivity decreases with the temperature and metals are PTC sensors.
- **Semiconductors**:
  - Intrinsic or lightly doped: the density of electrons and holes increases with the temperature as an exponential function. dn/dT and dp/dT $\gg$ $d\mu_{n,p}/dT$, thus, semiconductors are NTC sensors.
  - Doped semiconductors, the density of mobile charges is defined by the dopants concentration. Then, doped semiconductors are PTC sensors while devices based on intrinsic or lightly doped semiconductors behaves as NTC sensors.

Metals and doped semiconductors

$\mu(T) \Downarrow T$

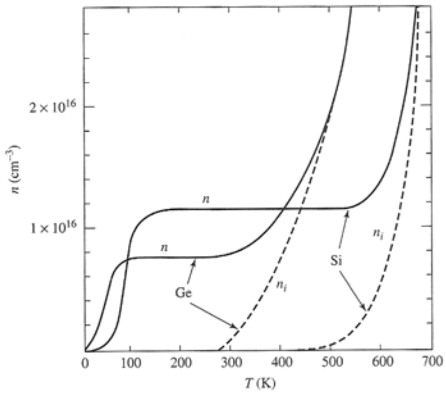
$n = \text{constant}$ $\Rightarrow$ **PTC**

Intrinsic semiconductors

$\mu(T) \Downarrow T$

$n \Uparrow\Uparrow T$ $\Rightarrow$ **NTC**

Electron density vs. temperature (example: silicon)



$$doped: n = N_D;\ p = \frac{n_i^2}{N_D}$$

$$intrinsic: n_i = p_i$$

At T<500 K, in intrinsic or weakly doped semiconductors the density of carriers changes with the temperature.

in an intrinsic semiconductor:

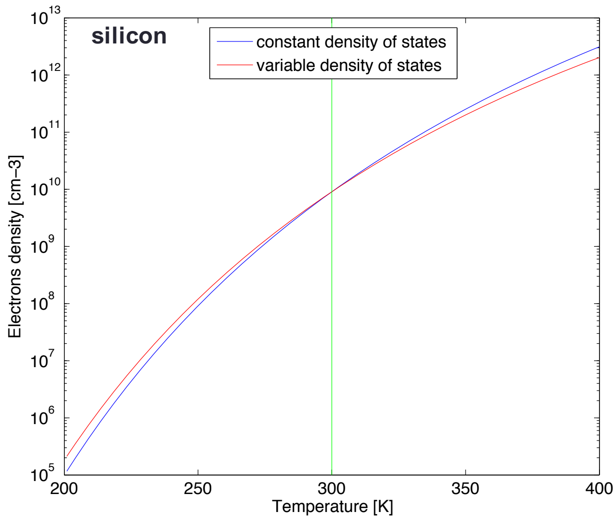
$$n = \sqrt{n_i^2} = \sqrt{N_c N_v} \cdot exp(-\frac{E_{gap}}{2kT})$$

the density of state is weakly dependent on T

$$N_C = 2\left(\frac{2\pi m_n^* kT}{\hbar^2}\right)^{\frac{3}{2}}$$

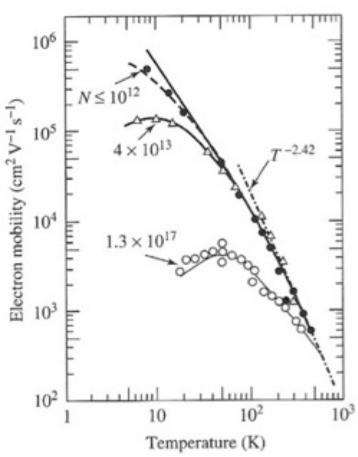
$$N_V = 2\left(\frac{2\pi m_p^* kT}{\hbar^2}\right)^{\frac{3}{2}}$$

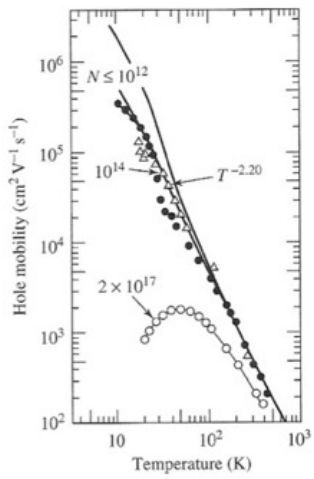
in practice, the exponential function dominates.



Mobility vs. temperature (the case of silicon)

The relationship between the mobility and the temperature depends on the concentration of the doping.





Electrons mobility

$$\mu_n = \mu_{0n} \cdot T^{-2.42}$$

$$@T = 300K \quad \mu_n = 2000\, cm^2 V^{-1} s^{-1} \Rightarrow$$

$$\mu_{0n} = \frac{\mu_{300}}{300^{-2.42}} = 1.97 \cdot 10^9\ cm^2 V^{-1} s^{-1}$$

$$\mu_n(T) = 1.97 \cdot 10^9 \cdot T^{-2.42}$$

Holes mobility

$$\mu_p = \mu_{0p} \cdot T^{-2.20}$$

$$@T = 300K \quad \mu_p = 500\, cm^2 V^{-1} s^{-1} \Rightarrow$$

$$\mu_{0p} = \frac{\mu_{300}}{300^{-2.20}} = 0.14 \cdot 10^9\ cm^2 V^{-1} s^{-1}$$

$$\mu_n(T) = 0.14 \cdot 10^9 \cdot T^{-2.20}$$

Semiconductor thermistors $R=f(T)$

let us consider the variation of mobility negligible respect to the variation of the electrons density.

$$R(T) = \rho(T)\frac{l}{A} = \frac{1}{q[\mu_n(T)+\mu_p(T)]n(T)}\frac{l}{A} \approx \frac{1}{q(\mu_n+\mu_p)}\frac{1}{\sqrt{N_cN_v}exp(-\frac{E_{gap}}{2kT})}\frac{l}{A}$$

$$\rightarrow R(T) = K\cdot exp(\frac{B}{T})$$

$$con\ B = \frac{E_{gap}}{2k};\quad K = \frac{1}{q(\mu_n+\mu_p)}\frac{1}{\sqrt{N_cN_v}}\frac{l}{A}$$

the resistence is known at the reference temperature: T_0

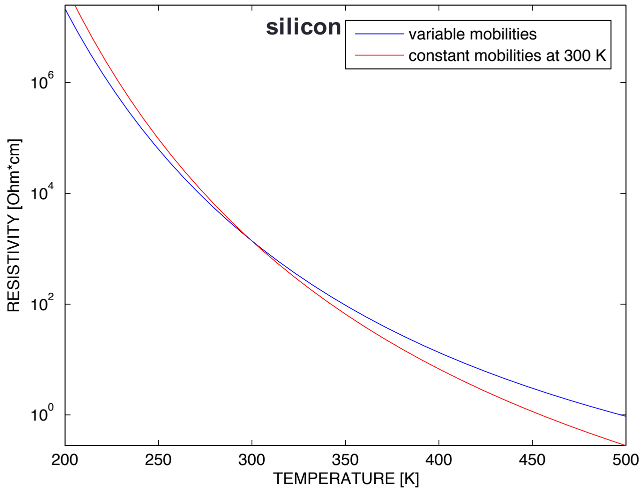
$$R(T_0) = K\cdot exp\left(\frac{B}{T_0}\right) \Rightarrow K = R(T_0)\cdot exp\left(-\frac{B}{T_0}\right)$$

$$R(T) = R(T_0)\cdot exp\left(-\frac{B}{T_0}\right)\cdot exp\left(\frac{B}{T}\right)$$

$$R(T) = R(T_0)\cdot exp\left(\frac{B}{T}-\frac{B}{T_0}\right) \qquad B = \frac{E_G}{2k}$$

$$R(T) = R(T_0)\cdot exp\left[B\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right]$$

$$T = \left[\frac{1}{T_0}+\frac{1}{B}\cdot ln\left(\frac{R(T)}{R_0}\right)\right]^{-1}$$



Linearized characteristics

- To calculate small changes around a fixed temperature, T_A, it is convenient to replace the thermistor characteristics with its linear approximation.

$$R(T) = R(T_0)\cdot exp\left[B\left(\frac{1}{T}-\frac{1}{T_0}\right)\right]$$

$$R(T) = R(T_A) + \left.\frac{dR}{dT}\right|_{T=T_A}\cdot(T-T_A) =$$

$$= R(T_A) + \left[-R(T_0)\cdot\frac{B}{T^2}\cdot exp B\left(\frac{1}{T}-\frac{1}{T_0}\right)\right]_{T=T_A}\cdot(T-T_A)$$

$$= R(T_A) - R(T_A)\cdot\frac{B}{T_A^2}\cdot(T-T_A)$$

$$\Rightarrow R(T) = R(T_A)\cdot\left[1-\frac{B}{T_A^2}\cdot(T-T_A)\right]$$

$$R(T) = R_0\cdot\left[1-\frac{B}{T_i^2}\cdot(T-T_0)\right] \quad\Rightarrow\quad R(T) = R_0\cdot[1+\alpha\cdot(T-T_0)]$$

$$\alpha = \frac{1}{R_0}\frac{dR}{dT} = \frac{d}{dT}\frac{R}{R_0} = -\frac{B}{T^2}$$

α is the temperature coefficient.
typical values around -0.05 K^{-1}

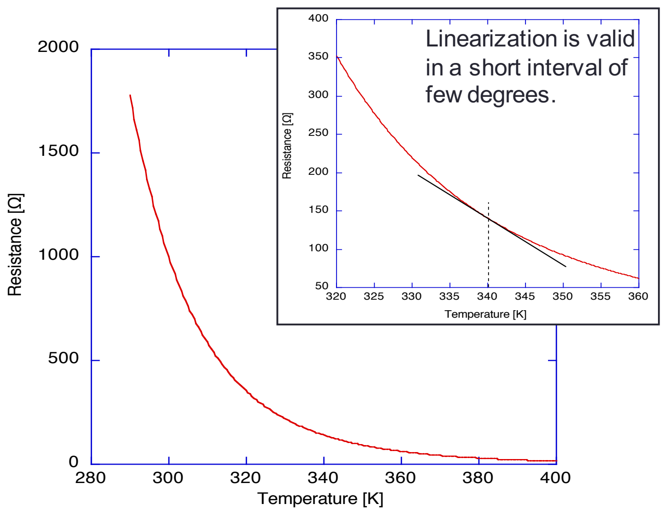
numerical example:

$B = 5000K;$

$T_A = 340K;\ R(T_A) = 140\Omega$

$$R(T) = 140\cdot\left[1-\frac{5000}{340^2}\cdot(T-340)\right] = 140\cdot[1-0.04\cdot(T-340)]$$

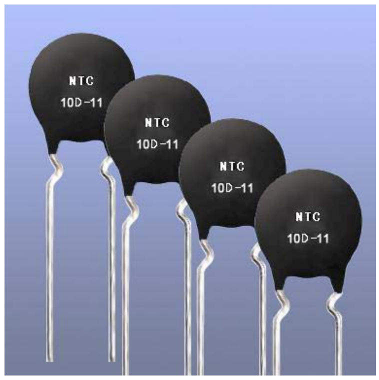
$$R(T) = R(T_A)\cdot[1-\alpha\cdot(T-T_A)]$$

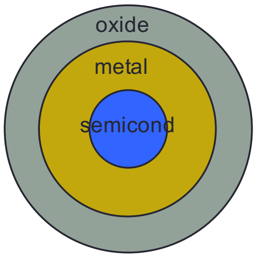


Thermistors

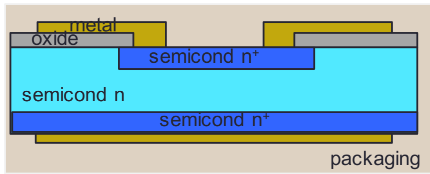
- The packaging is a suitable resin ensuring the electric insulation but the thermal conduction.
- A circular metallic contact is used in order to enhance the thermal contact of the semiconductor with the environment.
- Low cost thermistors are typically done with metal oxide semiconductors (e.g. SnO_2). The band gap of these materials is larger respect to silicon, and they can be used up to 500°C
  - At such temperature the conductivity of these materials is also sensitive to gas molecules.
  - Due to defects, the response curve of these materials may be a little different from that previously derived, in particular B may depend on T.
- The materials are lightly doped to increase the conductivity and also to maintain the density of carriers sensitive to temperature.
- The characteristics of less pure materias may deviate from the "silicon case":

$$T = \left[\alpha + \beta \cdot \left(ln\frac{R(T)}{R_0}\right) + \gamma \cdot \left(ln\frac{R(T)}{R_0}\right)^2 + \delta \cdot \left(ln\frac{R(T)}{R_0}\right)^3\right]^{-1}$$





Top view

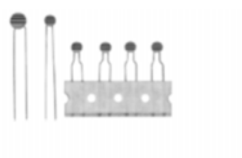


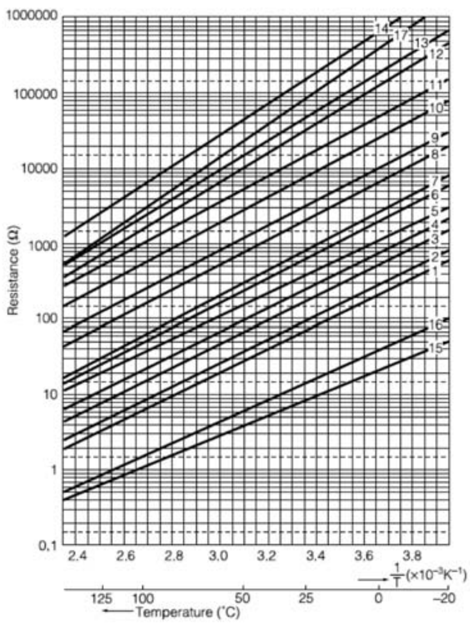
Side view

Examples of NTC thermistors

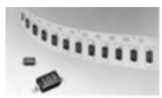
$$R(T) = R_0 exp(B(\frac{1}{T} - \frac{1}{T_0})$$
$$ln(\frac{R(T)}{R_0}) = B(\frac{1}{T} - \frac{1}{T_0})$$
$$log(\frac{R(T)}{R_0}) = \frac{B}{2.303 \cdot 1000}(\frac{1000}{T} - \frac{1000}{T_0})$$

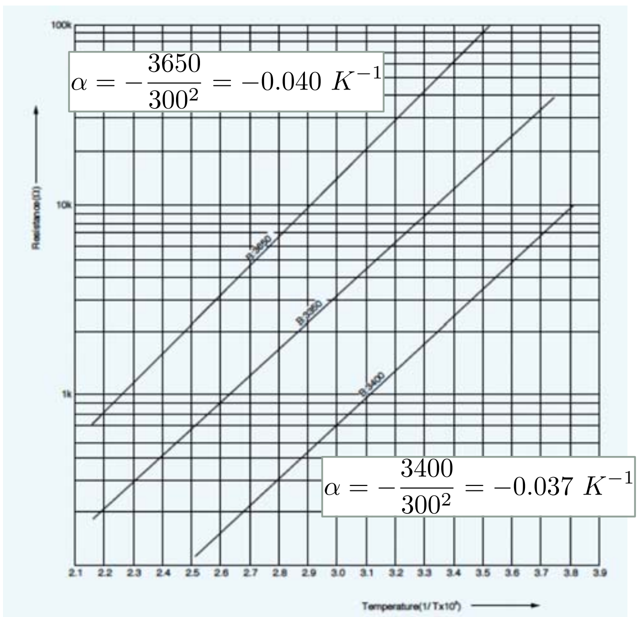
Bead sensor
Panasonic ERTD





Chip sensor
Semitec H3





$$\alpha = -\frac{3650}{300^2} = -0.040\ K^{-1}$$

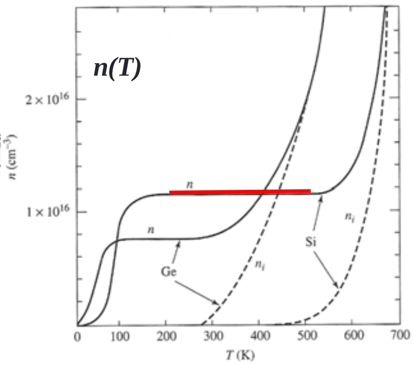
$$\alpha = -\frac{3400}{300^2} = -0.037\ K^{-1}$$

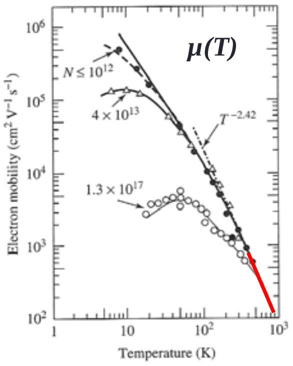
PTC semiconductor thermistors

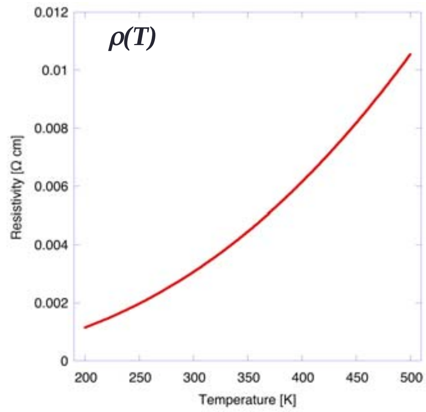
- In doped semiconductors, the density of charge carriers is independent from the temperature until the intrinsic charges are less than doping (n_i<N_D)
- In this case, the sensitivity is only provided by the mobility, and the sensor has a PTC behaviour.

$$\rho(T) = \frac{1}{q \cdot \mu(T) \cdot n(T)} = \frac{1}{q \cdot \mu_0 \cdot T^{-2.42} N_D} \Rightarrow \rho(T) = 0.31 \cdot 10^{-8} \cdot T^{2.42}\ \Omega \cdot cm$$

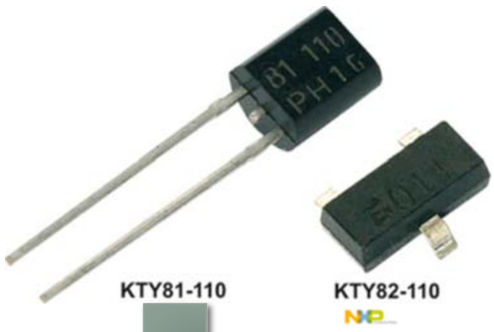
$$N_D = 10^{18}\ cm^{-3};\ \mu_0 = 1.97 \cdot 10^{9}\ \frac{cm^2}{V \cdot s \cdot K^{2.42}}$$



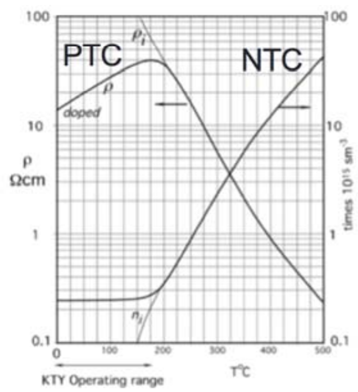


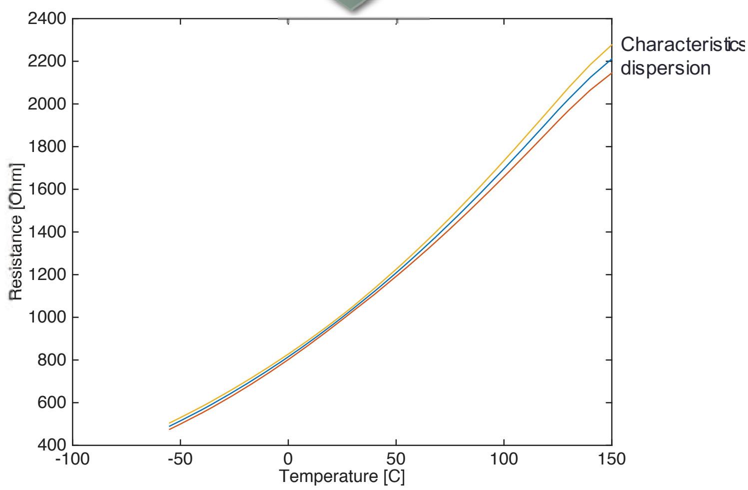


Example:
PTC thermistor Philips KTY81



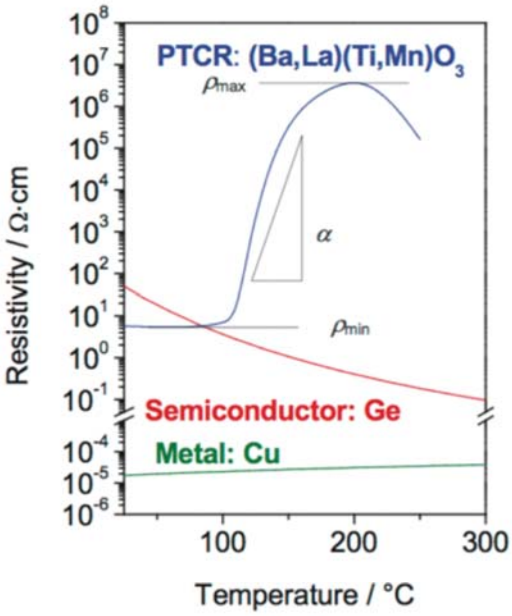
resisitiviy and charge carrier vs. temperature

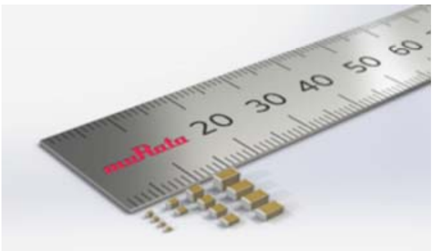


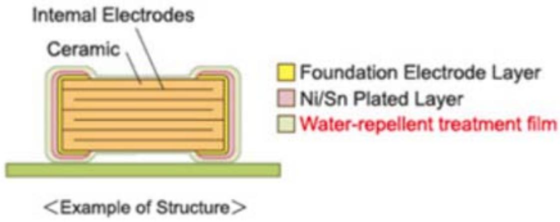


ceramic PTC thermistors

- Ceramic materials such as $BaTiO_3$ (barium titanate) undergo a phase transition at a specific temperature (Curie temperature).
- the electrical conductivity of $BaTiO_3$ can be settled, changing the materials composition and the fabrication process, in a wide range of orders of magnitude, from insulating to semiconducting behavior.
- The resistence is stable, but at the Curie temperature it abruptly increases giving rise to a sort of resistance switch that is used as a thermal protection in electronic circuits.



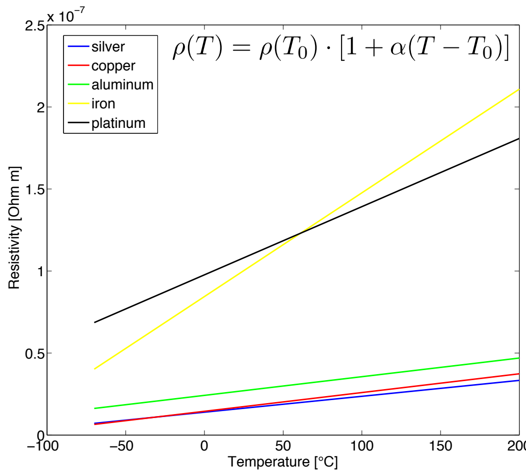




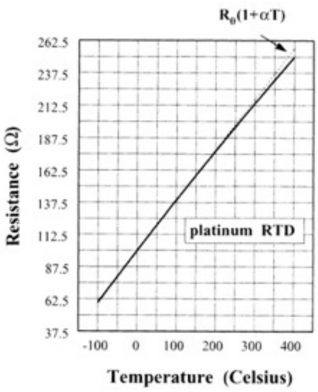
Resistance Temperature Detector (RTD)

- Metals (typically Pt, Cu, Ni,…)
- The resistance – temperature relationship depends only the mobility.

- General characteristics:
  - Good stability
  - Good reproducibility
  - Small non linearity
  - Variable dimensions and shapes



PT100



$$R(T) = R(T_0)\cdot\left[1+\alpha\cdot(T-T_0)\right]$$

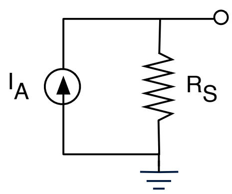
$$T_0 = 0°C \quad R(T_0) = 100\ \Omega$$

$$\alpha = 0.0039\ K^{-1}$$

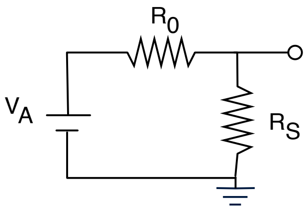
$$R(T) = 100\cdot\left[1+0.0039\cdot T\right] \ \text{con } T \text{ in } °C$$

Circuits for resistive sensors

- Resistive sensors are the most simple circuital elements.
- However, the optimization of circuit interface is not straightforward and it depends on the sensor feature that has to be optimized: sensitivity or linearity.
- The actual measured quantity is always a voltage.
  - Then, the most simple approach is to bias the sensor with a current source and measure the output voltage
  - The other approach, apparently more complicated, is based on voltage dividers



$$V_0(T) = I_A \cdot R_S(T)$$



$$V_0(T) = V_A \cdot \frac{R_S(T)}{R_0 + R_S(T)}$$

- In any case the sensitivity is the product of the circuit and the sensor.

$$R_S = R_0 + \Delta R = R_0 \cdot \left(1 + \frac{\Delta R}{R_0}\right) = R_0 \cdot (1 + \delta)$$

$$\delta_{thermistor} = exp\left[B\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] - 1$$

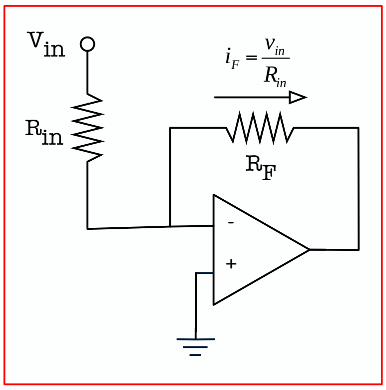
$$\delta_{RTD} = \alpha \cdot (T - T_0)$$

$$S = \frac{dV_0}{dT} = \boxed{\frac{dV_0}{d\delta}}\frac{d\delta}{dT}$$

Property of the circuit
It does not depend on sensors and measurand

Current source with op-amps

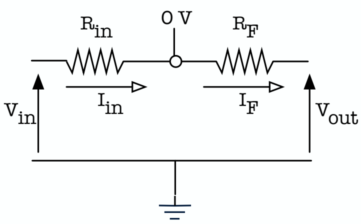
- The current delivered by an ideal current source is independent from the impedance of the load.
- Op-amps can provide the conditions for current sources.
- In a simple circuit, such as the inverting amplifier, the current in the feedback network does not depend on the feedback impedance, then the the circuit acts as a current source for the resistor R_F
  - In feedback configuration, op-amps delivers the output voltage necessary to maintain at zero the differential voltage across the input terminals.
- The non-ideality of the op-amp limits the ideality of the current source.



Inverting amplifier: equivalent circuit

$$A = -\frac{V_{out}}{V_+ - V_-}$$

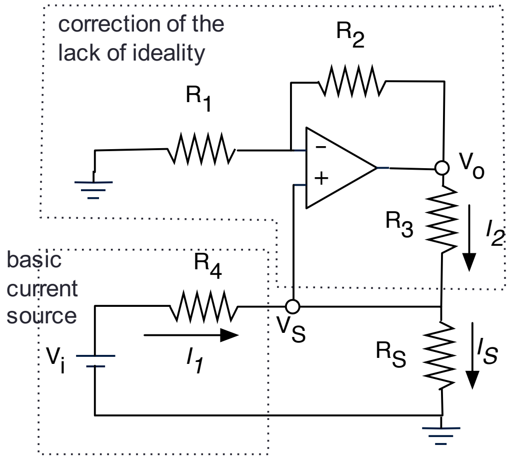
$$A = \infty \ e \ V_{out} \ finite \Rightarrow V_+ = V_-$$



$$I_{in} = \frac{V_{in}}{R_{in}} = I_F ; \frac{V_{in} - 0}{R_{in}} = \frac{0 - V_{out}}{R_F}$$

$$V_{out} = -\frac{R_F}{R_{in}} \cdot V_{in} = -I_{in} \cdot R_F$$

Howland current source: example1



- Useful for grounded sensors.
- The lack of ideality of the voltage divider is complemented by the additional current provided by an op-amp.
- A simple implementation of the idea is achieved with a non-inverting amplifier.

$$R_1 = R_2 \; ; \; R_3 = R_4 = R$$

$$V_0 = V_S \cdot \left(1 + \frac{R_2}{R_1}\right) = 2 \cdot V_S$$

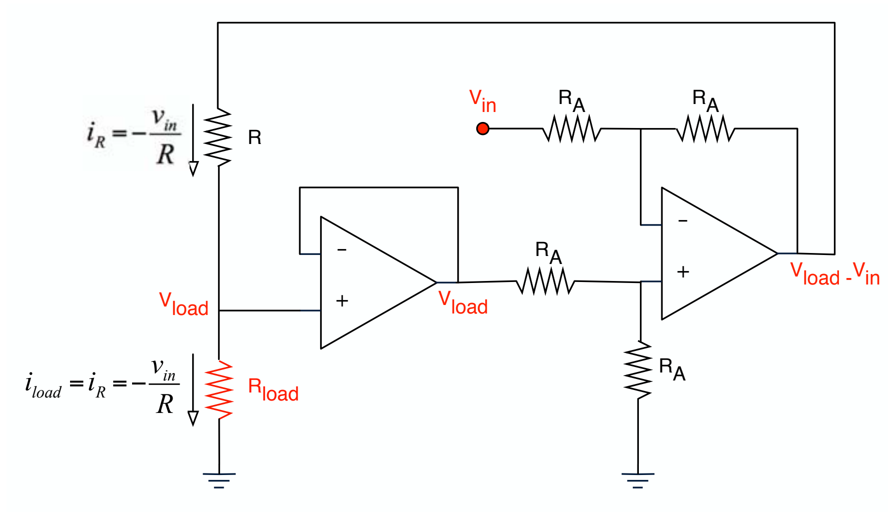
$$I_S = I_1 + I_2 = \frac{V_i - V_S}{R_4} + \frac{2 \cdot V_S - V_S}{R_3} = \frac{V_i - V_S}{R} + \frac{V_S}{R} = \frac{V_i}{R}$$

Howland current source: example 2

- Another implementation of the concept: the output of the differential amplifier (ideal op-amp) is V_{load}-V_{in}
- The current in the resistor R is

$$i_R = \frac{(v_{load} - v_{in}) - v_{load}}{R} = -\frac{v_{in}}{R}$$

- Since the input impedance of the ideal op-amp is infinite, the current I_R is totally injected in the load, and then it does not depend on the load.

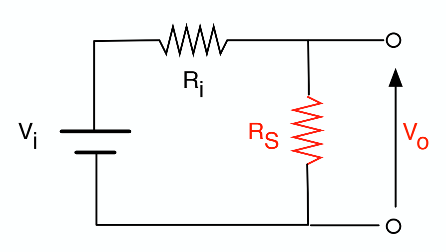


Measure of a resistive sensor with a voltage divider

- The voltage divider is the more general circuit available to extract a signal from a resistive sensor

$$\Delta M \rightarrow \Delta R \rightarrow \Delta V \quad R = f(M); \quad V_0 = f(R)$$

$$R_S = R_0 + \Delta R = R_0 \cdot \left(1 + \frac{\Delta R}{R_0}\right) = R_0 \cdot (1 + \delta)$$



- The relationship between the output and the variation of R_S is non-linear.

$$V_o = V_i \cdot \frac{R_o \cdot (1+\delta)}{R_i + R_o \cdot (1+\delta)}$$

- Then, even with a linear sensor, the voltage divider gives rise to a non linear relationship between the signal and the measurand.
- R_i can be chosen to optimize either the sensitivity or the linearity

Sensitivity optimization

- The total sensitivity is the product of the sensitivity of the sensor and the sensitivity of the circuit.
- The sensitivity of the circuit is independent from the nature of the sensor.
- To optimize the contribution of the circuit, let us maximize dV/dδ.
- Since the relationship V=f(δ) is non linear, the sensitivity depends on δ.
- Let us choose δ=0. In this way, the LOD is optimized.

$$S = \frac{dV}{dM} = \frac{dV}{d\delta} \cdot \frac{d\delta}{dM}$$

$$V_o = V \cdot \frac{R_o \cdot (1+\delta)}{R_i + R_o \cdot (1+\delta)}$$

$$S = \frac{dV}{d\delta} = V \frac{R_o \cdot [R_i + R_o \cdot (1+\delta)] - R_o \cdot [R_o \cdot (1+\delta)]}{[R_i + R_o \cdot (1+\delta)]^2} = V \frac{R_o \cdot R_i}{[R_i + R_o \cdot (1+\delta)]^2}$$

$$max\, S \Rightarrow \frac{dS}{dR_i} = 0$$

$$\frac{dS}{dR} = V \frac{R_o \cdot [R_i + R_o \cdot (1+\delta)]^2 - R_o \cdot R_i \cdot 2 \cdot [R_i + R_o \cdot (1+\delta)]}{[R_i + R_o \cdot (1+\delta)]^4}$$

$$\frac{dS}{dR} = 0 \Rightarrow R_o \cdot [R_i + R_o \cdot (1+\delta)]^2 - R_o \cdot R_i \cdot 2 \cdot [R_i + R_o \cdot (1+\delta)] = 0$$

$$R_o \cdot [R_i + R_o \cdot (1+\delta)] = R_o \cdot R_i \cdot 2; \; R_i + R_o \cdot (1+\delta) = R_i \cdot 2;$$

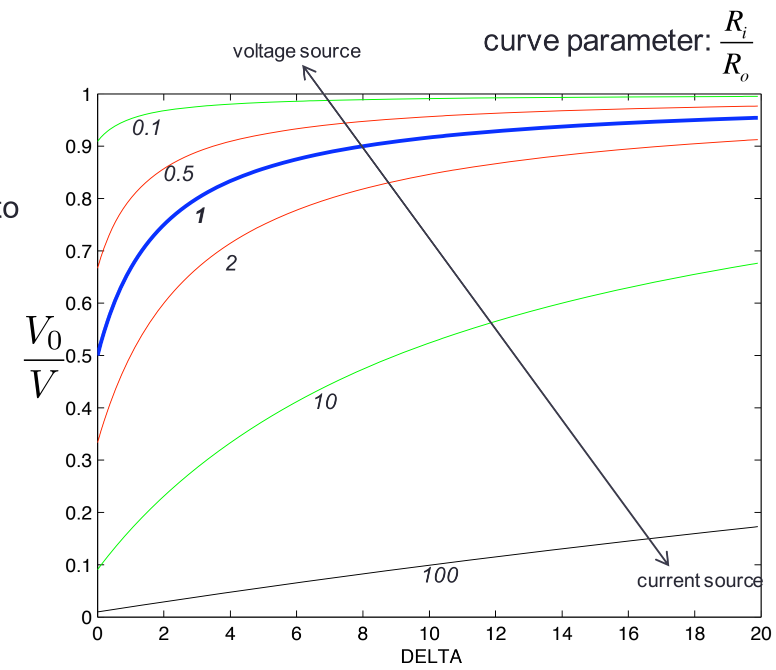
$$R_o + R_o \cdot \delta = R_i$$

the maximum sensitivity around δ=0 is obtained when: $R_i = R_o$

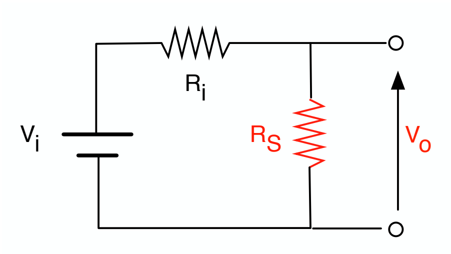
Linearity and sensitivity

- The ratio R_i/R_0 affects the sensitivity and the linearity of the signal respect to the measurand
- Sensitivity and linearity are inversely correlated.

$$V_o = V \cdot \frac{R_0 \cdot (1+\delta)}{R_i + R_o \cdot (1+\delta)} \Rightarrow \frac{V_o}{V} = \frac{(1+\delta)}{\frac{R_i}{R_o} + (1+\delta)}$$



Behaviour around δ=0



$$V_o = V \cdot \frac{R_o \cdot (1+\delta)}{R_o + R_o \cdot (1+\delta)} = V \cdot \frac{1+\delta}{2+\delta}$$

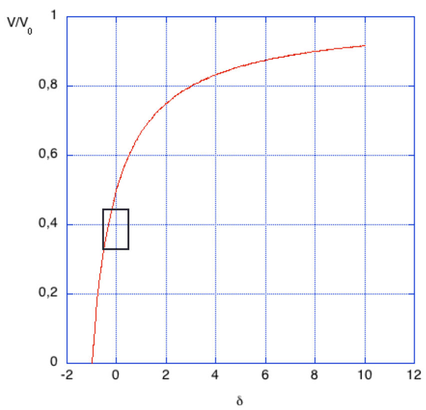
linearization around $\delta = 0$

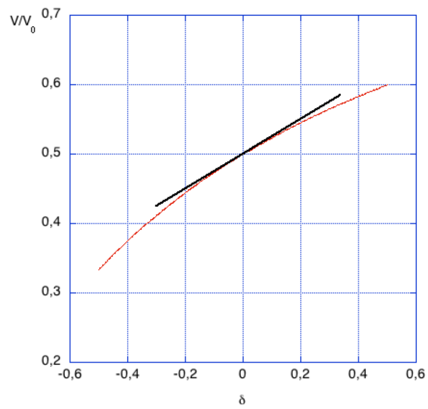
$$\text{when } \delta << 1 \Rightarrow V_o(\delta) = V_o(\delta = 0) + \left.\frac{dV_o}{d\delta}\right|_{\delta=0} \cdot \delta$$

$$V_o(\delta) = \frac{V}{2} + V \left.\frac{1}{(2+\delta)^2}\right|_{\delta=0} \cdot \delta$$

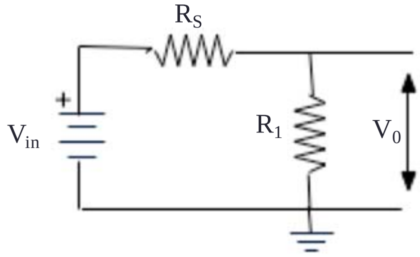
$$V_o(\delta) = \frac{V}{2} + \frac{V}{4} \cdot \delta$$

$$S = \frac{dV_o}{d\delta} = \frac{V}{4}$$





example:
NTC Thermistor + Voltage divider + Arduino

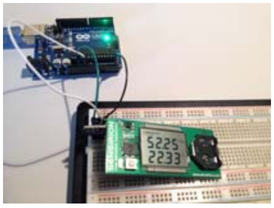


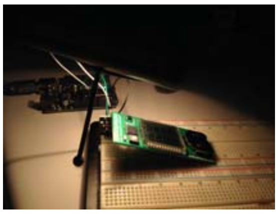
$$V_{in} = 5\ V; R_1 = 10\ K\Omega$$

$$V_0 = V_{in}\frac{R_1}{R_1 + R_S} \rightarrow R_S = R_1\frac{V_{in} - V_0}{V_{in}}$$

$$R_S = R_0 * exp(B \cdot (\frac{1}{T} - \frac{1}{T_0}))$$

calibration:
heat source: halogen lamp; calibrator: integrated sensor (Sensirion SHT21)



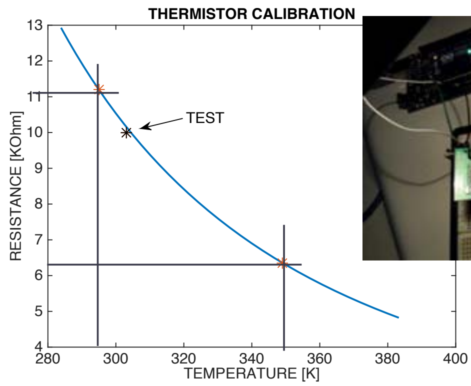


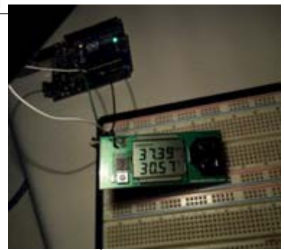
$$T_1 = 22 + 273 = 293\ K \rightarrow R_s = 11.2\ K\Omega$$
$$T_2 = 76 + 273 = 349\ K \rightarrow R_s = 6.35\ K\Omega$$

$$B = 1079\ K;$$
$$T_0 = 293\ K;$$
$$R_0 = 11.20\ K\Omega$$

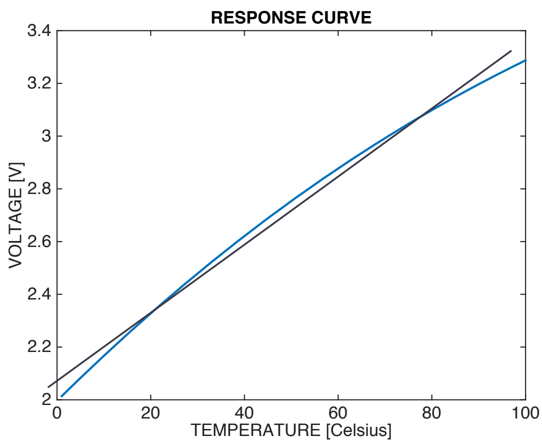
$$T_{test} = 37 + 273 = 310\ K$$
$$R_{text} = 9\ K\Omega$$

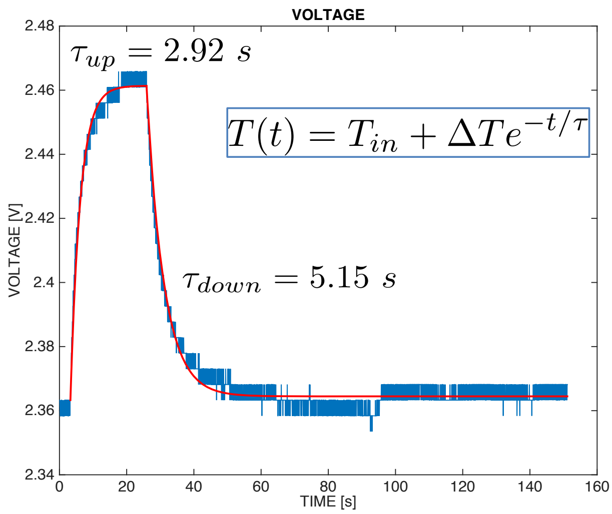
$$T = \frac{1}{\frac{1}{B}log\left(\frac{R_s}{R_0}\right) + \frac{1}{T_0}} =$$
$$= 311.4\ K = 38.4\ K$$

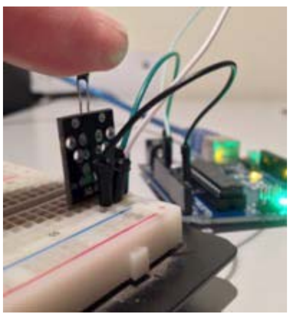


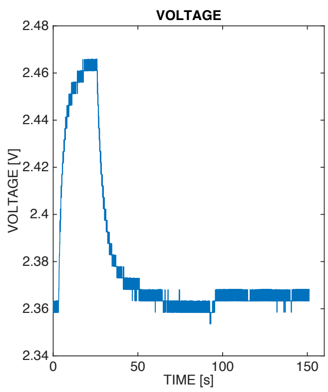


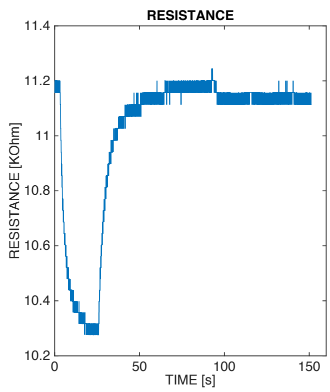
$$V_0 = V_i\frac{R_1}{R_1 + R_0 exp(B(\frac{1}{T} - \frac{1}{T_0})}$$

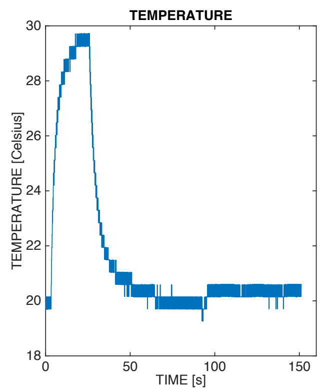


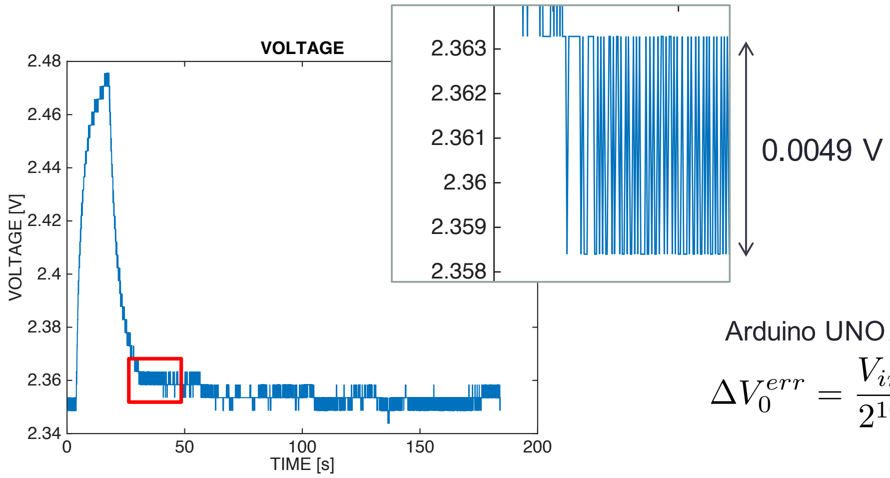












Arduino UNO ADC resolution: 10 bit

$$\Delta V_0^{err} = \frac{V_{in}}{2^{10}} = \frac{5}{1024} = 0.0049\ V$$

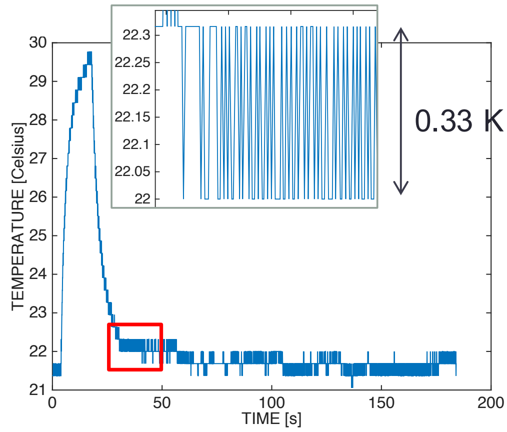
Resolution

$$T = 22°C$$

$$S = \frac{dV_0}{dT} = -R_1 V_{in} \frac{\alpha R_0}{R_1 + R_0(1+\alpha(T-T_0))^2} \quad (\alpha = -\frac{B}{T^2})$$

$$S_{22} = -0.0349 \frac{V}{K}$$

$$T_{res} = \frac{\Delta V_0^{err}}{S_{22}} = \frac{0.0049}{0.0349} = 0.33\ K$$



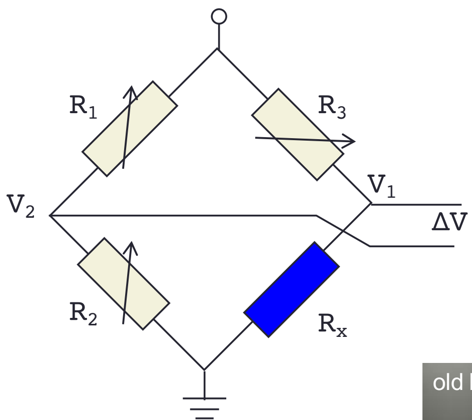
Wheatstone bridge

- Circuit used to measure an unknown resistor through a null measurement.
- The null condition ($\Delta V=0$) is achieved changing the value of three known resistors.

$$V_1 = V\frac{R_x}{R_3+R_x} \qquad V_2 = V\frac{R_2}{R_1+R_2}$$

$$\Delta V = V_1 - V_2 = V\cdot\left(\frac{R_x}{R_3+R_x} - \frac{R_2}{R_1+R_2}\right) = 0$$

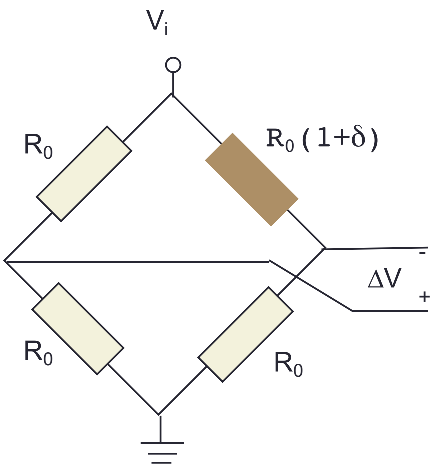
$$\frac{R_x}{R_3+R_x} - \frac{R_2}{R_1+R_2} = \frac{1}{\frac{R_3}{R_x}+1} - \frac{1}{\frac{R_1}{R_2}+1} = 0 \Rightarrow \frac{R_3}{R_x} = \frac{R_1}{R_2} \Rightarrow R_x = \frac{R_3 \cdot R_1}{R_2}$$



modern box of resistors with R variable in the range 0-12 KΩ, with 6 decades of steps of 10 mΩ each.

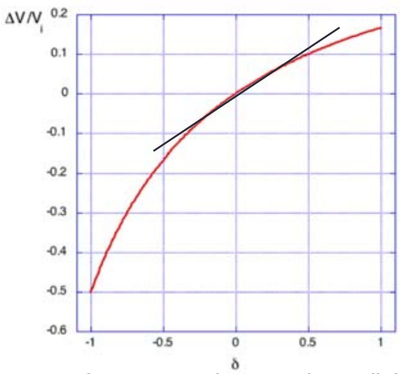
Application of the Wheatstone bridge to the measure of a resistive sensor

- 3 fixed resistances and one resistive sensor
- The fixed resistors are chosen in order to balance the bridge when the measurand is null.



$$R_S = R_o + \Delta R_o = R_o \cdot \left(1 + \frac{\Delta R_o}{R_o}\right) = R_o \cdot (1+\delta)$$

$$\Delta V = \frac{V}{2} - V\frac{R_0}{R_0 + R_0 \cdot (1+\delta)} = \frac{V}{2} - \frac{V}{2+\delta} = \frac{V}{2}\frac{\delta}{(2+\delta)} = \frac{V}{4}\frac{\delta}{1+\frac{\delta}{2}}$$

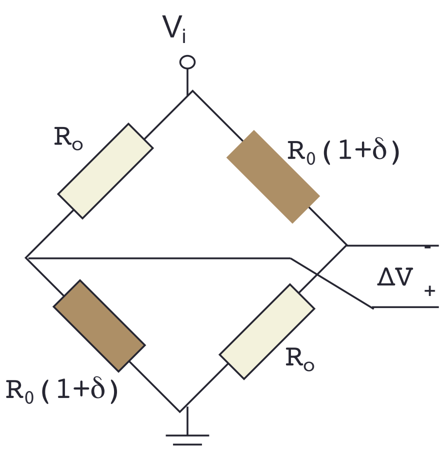


$$if \quad \delta \ll 1 \Rightarrow \Delta V = \frac{V}{4}\delta$$

it corresponds to a voltage divider with the offset subtraction. Respect to the voltage divider it exploits the whole range provided by the voltage source.

2 identical sensors exposed to the same measurand

- The sensitivity increases using two identical sensors exposed to the same measurand.
- The sensors are connected in different legs and in opposite positions.



$$\Delta V = V_i \cdot \frac{R_0 \cdot (1+\delta)}{R_0 + R_0 \cdot (1+\delta)} - V\frac{R_0}{R_0 + R_0 \cdot (1+\delta)} =$$

$$= V \cdot \left(\frac{1+\delta}{2+\delta} - \frac{1}{2+\delta}\right) = V \cdot \frac{\delta}{2+\delta}$$

For small variation of the sensor response

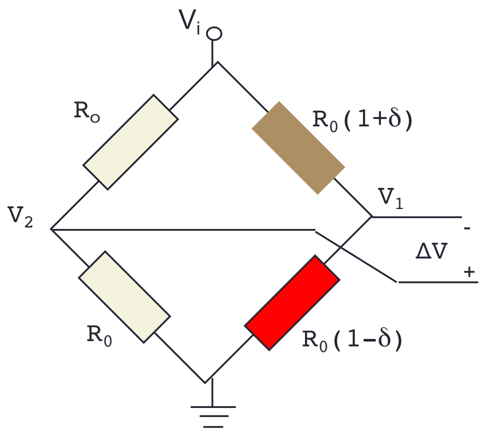
$$\delta \ll 1 \Rightarrow \delta \ll 2 \Rightarrow \Delta V = \frac{V}{2}\delta$$

$$S = \frac{V}{2}$$

The sensitivity is twice the sensitivity with one sensor

2 identical sensors exposed to opposite changes

- The use of identical sensors exposed to opposite variations allows to fully exploit the bridge features
  - this case is appliable to some experimental conditions such as the measurement of the deformation of a beam or a cantilever.
- The sensors are connected in the same leg.



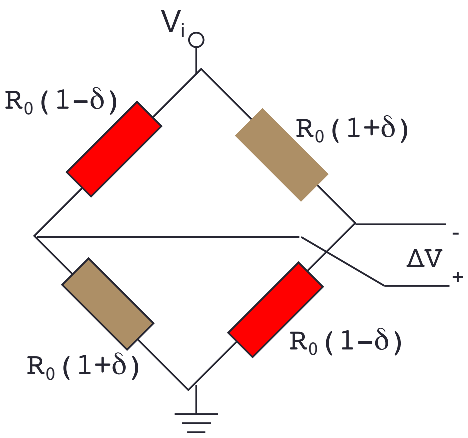
$$\Delta V = V_i \cdot \frac{R_0}{2\cdot R_0} - V_i \cdot \frac{R_0\cdot(1-\delta)}{R_0\cdot(1-\delta)+R_0\cdot(1+\delta)} =$$

$$= \frac{V_i\cdot}{2} - V_i \cdot \frac{1-\delta}{2} = \frac{V_i\cdot}{2}\cdot\delta \quad \text{linear !}$$

$$S = \frac{d\Delta V}{d\delta} = \frac{V_i}{2}$$

the same of the sensitivity in the origin in the case with 2 sensors exposed to same stimulus

2 pairs of sensors: same and opposite stimuli



$$\Delta V = V_i \cdot \frac{R_0\cdot(1+\delta)}{R_0\cdot(1+\delta)+R_0\cdot(1-\delta)} - V_i \cdot \frac{R_0\cdot(1-\delta)}{R_0\cdot(1+\delta)+R_0\cdot(1-\delta)} = V_i\cdot\delta$$

linear !

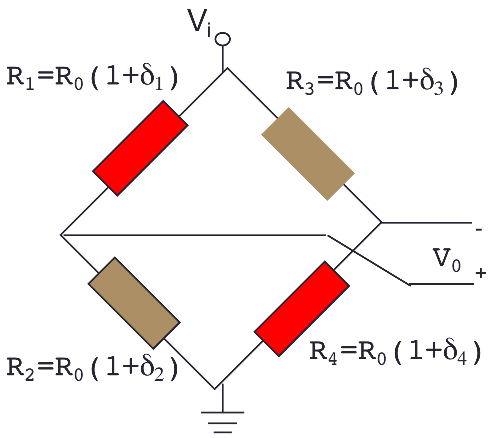
$$S = \frac{d\Delta V}{d\delta} = 1\cdot V_i$$

The sensitivity is four times the sensitivity with one sensor and twice the sensitivity with two sensors.

General case with 4 identical sensors

- Let us consider 4 sensors with the same null resistance but each undergoing a different relative change of resistance

$$R_i = R_0 \cdot (1+\delta_i) \quad i = 1-4$$



$$\frac{V_o}{V_i} = \frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}$$

$$\frac{V_o}{V_i} = \frac{R_0(1+\delta_2)}{R_0(1+\delta_1)+R_0(1+\delta_2)} - \frac{R_0(1+\delta_4)}{R_0(1+\delta_3)+R_0(1+\delta_4)}$$

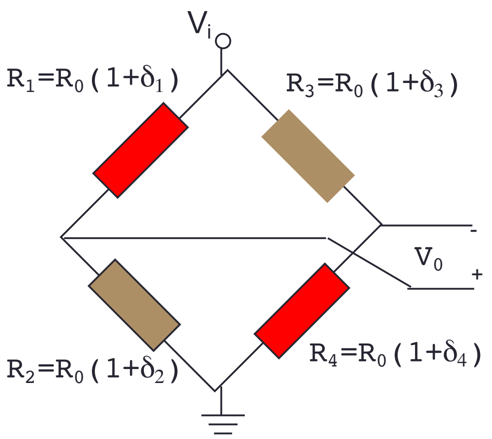
If $\delta \ll 1$, the second order terms (δ^2, $\delta_i\delta_j$) are negligible, then the following expression is found:

$$\frac{V_o}{V_i} = \frac{1}{4}(+\delta_1 - \delta_2 - \delta_3 + \delta_4)$$

This property is useful to compensate cross-interferences.

Proof

Let s write: $R_i = R_0 \cdot (1+\delta_i) = R_0 + \Delta R_i$



$$\frac{V_o}{V_i} = \frac{R+\Delta R_4}{R+\Delta R_3+R+\Delta R_4} - \frac{R+\Delta R_2}{R+\Delta R_1+R+\Delta R_2}$$

$$\left(assumption: \Delta R_i \ll R \; e \; \Delta R_i \Delta R_j = 0\right)$$

numerator:

$$(R+\Delta R_4)\cdot(R+\Delta R_1+R+\Delta R_2)-(R+\Delta R_2)\cdot(R+\Delta R_3+R+\Delta R_4)$$
$$\cong R\cdot(+\Delta R_1-\Delta R_2-\Delta R_3+\Delta R_4)$$

denominator:

$$(R+\Delta R_3+R+\Delta R_4)\cdot(R+\Delta R_1+R+\Delta R_2)$$
$$= 4\cdot R^2 + 2\cdot R\cdot(\Delta R_1+\Delta R_2+\Delta R_3+\Delta R_4) \cong 4\cdot R^2$$

$$\frac{V_o}{V_i} = \frac{R\cdot(+\Delta R_1-\Delta R_2-\Delta R_3+\Delta R_4)}{4\cdot R^2} = \frac{1}{4}\cdot\left(+\frac{\Delta R_1}{R}-\frac{\Delta R_2}{R}-\frac{\Delta R_3}{R}+\frac{\Delta R_4}{R}\right) =$$
$$\frac{1}{4}\cdot(+\delta_1-\delta_2-\delta_3+\delta_4)$$

Self-heating

- The resistance depends on the actual temperature of the thermistor.
- During the measurement, the thermistor is biased and then, due to Joule effect, the sensor temperature increases.
- The actual thermistor temperature is $T_A+\Delta T_{SH}$ where T_A is the temperature to be measured and ΔT_{SH} is the self-heating error.
- The thermal process leading to the actual thermistor temperature is ruled by the heat conservation law:

$$\Delta Q_{abs} = \Delta Q_{acq} - \Delta Q_{lost}$$

ΔQ_{acq}: heat acquired by Joule effect
ΔQ_{lost}: heat dissipated into the environment
ΔQ_{abs}: heat absorbed by the thermistor

- The increase of temperature is proportional to the absorbed heat and the constant of proportionality is the thermal capacity C=m c, where m is the mass and c is the specific heat.
- The self-heating process is universal for any resistive device.

Calculation of self-heating

$$\Delta Q_{abs} = \Delta Q_{acq} - \Delta Q_{lost};$$

$$dQ_{abs} = mc \cdot dT$$
$$dQ_{acq} = P_{el} \cdot dt = VI \cdot dt$$
$$dQ_{lost} = P_{diss} dt = \delta \cdot (T - T_A) \cdot dt$$

δ: thermal dissipation coefficient or thermal conductivity (W/K)
m: mass (Kg)
c: specific heat (J/Kg K)

$$mc \cdot dT = VI \cdot dt - \delta(T - T_A)$$

$$mc \frac{dT}{VI - \delta(T - T_A)} = dt$$

$$\int_{T_A}^{T} \frac{dT}{VI - \delta(T - T_A)} = \int_0^t \frac{dt}{mc}$$

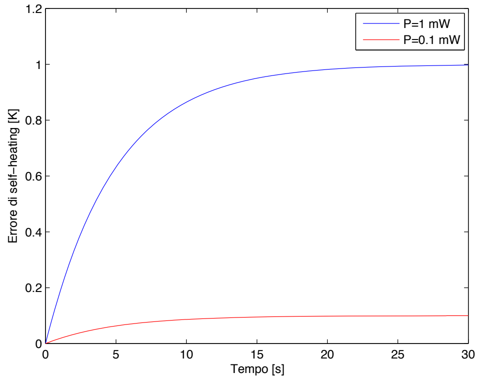
$$u = VI - \delta(T - T_A); du = -\delta T$$

$$-\int_{VI}^{VI-\delta(T-T_A)} \frac{du}{u} = -t\frac{\delta}{mc}$$

$$ln\frac{VI - \delta(T - T_A)}{VI} = -t\frac{\delta}{mc}$$

$$T - T_A = \frac{VI}{\delta}\left[1 - exp\left(-t\frac{\delta}{mc}\right)\right]$$

Example with R_0=5 KΩ and $\delta=10^{-3}$ W/K



$$P = 1\ mW \Rightarrow I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1\ mW}{5\ K\Omega}} = 0.44\ mA \quad e \quad V = R \cdot I = 2.23\ V$$

$$P = 0.1\ mW \Rightarrow I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.1\ mW}{5\ K\Omega}} = 0.31\ mA \quad e \quad V = R \cdot I = 1.58\ V$$

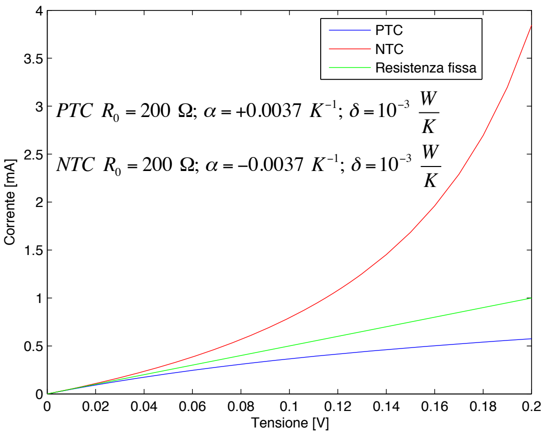
The self-heating can be reduced:
increasing δ
stirring the media (if liquid or gaseous) or coating the device with a resin with high thermal conductivity.
reducing the electric power

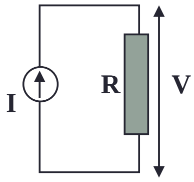
Self-heating and circuits stability

- As the current and the voltage increase, the electric power dissipated in a resistor increases.
- The self-heating changes the resistance according to its PTC or the NTC character
- NTC: the resistance decreases, and it acts as a negative feedback for current sources
- PTC: the resistance increases, and it acts as a negative feedback for voltage sources

$$R = R_0 \cdot \left[1 + \alpha \cdot \frac{V \cdot I}{\delta}\right]$$

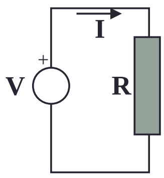
$$V = R \cdot I = R_0 \cdot I \cdot \left[1 + \alpha \cdot \frac{V \cdot I}{\delta}\right] = R_0 \cdot I + \frac{\alpha \cdot R_0 \cdot I^2}{\delta} \cdot V \Rightarrow V = R_0 \cdot I \cdot \frac{1}{1 - \alpha \cdot \frac{R_0 \cdot I^2}{\delta}}$$





NTC

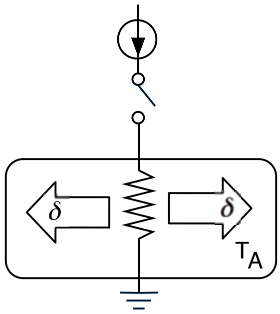
$I \uparrow \Rightarrow T_R \uparrow \Rightarrow R \downarrow \Rightarrow V = R \cdot I$ *stabilized*



PTC

$V \uparrow \Rightarrow T_R \uparrow \Rightarrow R \uparrow \Rightarrow I = \frac{V}{R}$ *stabilized*

Si consideri un termistore NTC la cui resistenza a T=0°C è pari a 20 KΩ e caratterizzato da un parametro B=3350 K. Si supponga di accoppiare termicamente il sensore ad un corpo di temperatura T=50°C e sia $\delta = 10^{-3}$ W/K il coefficiente di dissipazione termica. Si calcoli l'errore di self-heating quando il sensore è alimentato da un corrente di 1 mA.



la soluzione con la caratteristica esatta richiede la soluzione di una equazione non lineare.

$$\Delta T = \frac{R \cdot I^2}{\delta} \Rightarrow \Delta T = R_{273} \cdot exp\left(\frac{B}{T + \Delta T} - \frac{B}{273}\right) \cdot \frac{I^2}{\delta}$$

1. linearizzazione nell'intorno di T=50°C = 323 K

$$R_T = R_{T_0} \cdot exp\left(B \cdot \left(\frac{1}{T} - \frac{1}{T_0}\right)\right) = 20 \cdot exp\left(3350 \cdot \left(\frac{1}{323} - \frac{1}{273}\right)\right) = 2.992\ K\Omega$$

$$\alpha = -\frac{B}{T^2} = -\frac{3350}{323^2} = -0.032\ K^{-1}$$

2. variazione di temperatura dovuta al self-heating

$$\Delta T = \frac{R \cdot I^2}{\delta} = \frac{R_{323} \cdot (1 + \alpha \cdot \Delta T) \cdot I^2}{\delta}$$

da cui :

$$\Delta T = \frac{\frac{R_{323} \cdot I^2}{\delta}}{1 - \alpha \cdot \frac{R_{323} \cdot I^2}{\delta}} = \frac{\frac{2992 \cdot 10^{-2}}{10^{-3}}}{1 + 0.032 \cdot \frac{2992 \cdot 10^{-2}}{10^{-3}}} = 2.7\ K$$

3. verifica della approssimazione

confronto del valore di R(T+ΔT) con la caratteristica esatta ed approssimata

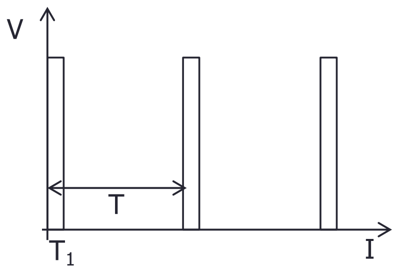
Esatta: $R^{es}_{323+2.7} = 20 \cdot exp\left(3350 \cdot \left(\frac{1}{323 + 2.7} - \frac{1}{273}\right)\right) = 2.746\ K\Omega$

approssimata $R^{app}_{323+2.7} = 2.992 \cdot (1 - 0.032 \cdot 2.7) = 2.730 K\Omega$

Errore relativo: $\frac{R^{es} - R^{app}}{R^{es}} = 0.005$

Pulsed bias

- The self-heating can be reduced biasing the sensor with a pulsed bias
- The root mean square value of a a.c. signal is defined as the equivalent d.c. signal dissipating the same amount of power.
- Given two signals of same magnitude d.c. and pulsed, the self-heating induced by the pulsed signal is reduced of a factor equal to the square root of the duty cycle.



$$v_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T v^2(t) \cdot dt} = V_{max} \cdot \sqrt{\frac{T_1}{T}}$$

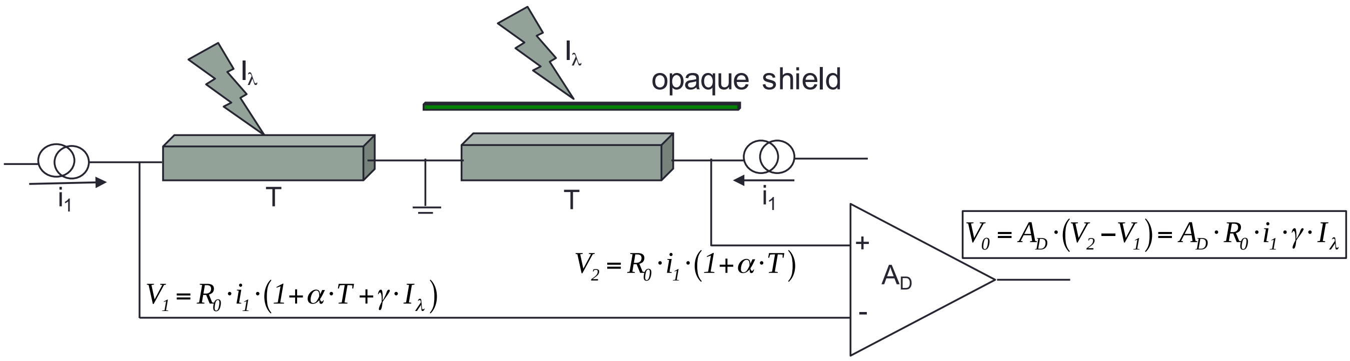
$$steady\ state\ temperature = \frac{V_{max}^2}{R\delta} \frac{T_1}{T}$$

Cross-sensitivity in semicond

- The conducitivty of semiconductor may be altered by several different quantities
  - temperature, radiation, magnetic field, mechanical stress, adsorbed gases....
- This is an example of non-selectivity
- Such sensors can be either used in combination with others (sensor array) or in arrangement among two or more sensors where common mode (undesired quantities) can be canceled.
- Example: A sensor sensitive to light and temperature

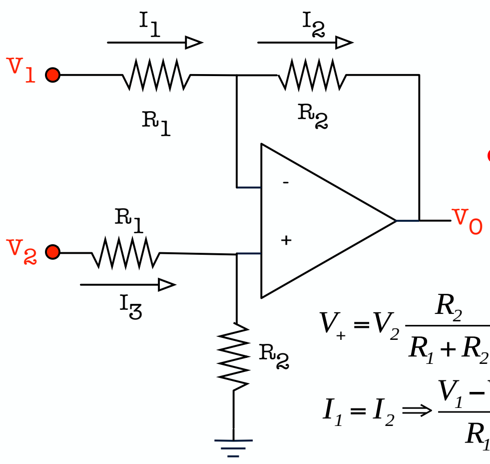
$$R = R_0 \cdot (1 + \alpha \cdot T + \gamma \cdot I_\lambda)$$

- A DIfferential amplifier cancels the dependence on the temperature and emphasise the dependence on the radiation.



Differential amplifier

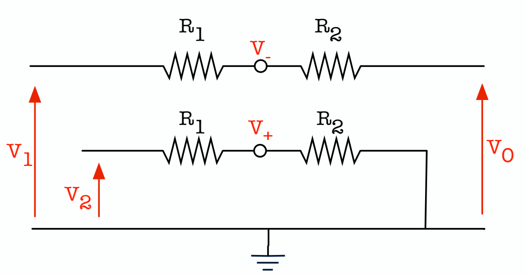
- A differential amplifier with infinite common mode rejection (CMRR) can be obtained with an ideal op-amp and resistors with accurate values.
- The gain depends on the ratio between R_2 and R_1. To change the gain and to maintain the CMRR it is necessary to change two pairs of resistors of exactly the same quantity.



op.amp. ➜ $V_+ = V_-$

$$V_+ = V_2 \frac{R_2}{R_1 + R_2}$$

$$I_1 = I_2 \Rightarrow \frac{V_1 - V_-}{R_1} = \frac{V_- - V_{out}}{R_2}$$



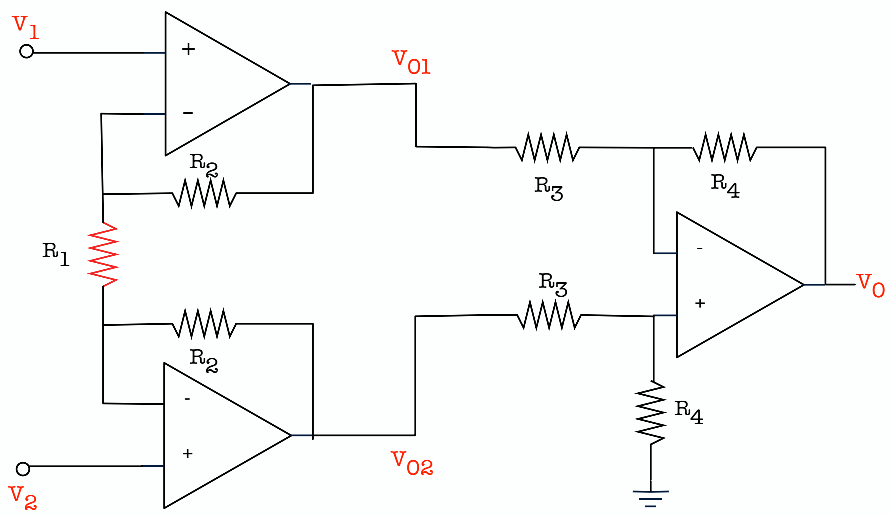
$$V_- = \frac{V_1 R_2 + V_{out} R_1}{R_1 + R_2}$$

$$V_+ = V_- \Rightarrow V_2 \frac{R_2}{R_1 + R_2} = \frac{V_1 R_2 + V_{out} R_1}{R_1 + R_2}$$

$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

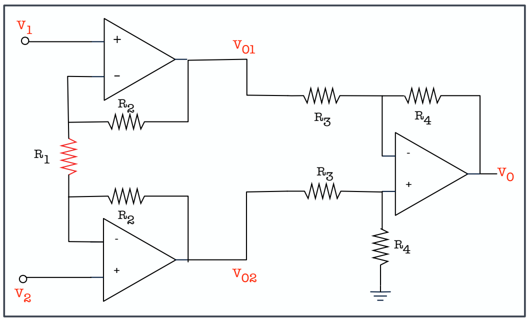
Instrumentation amplifier

- The instrumentation amplifier is a differential amplifier with a input stage.
- The gain can be adjusted changing only the resistor (R_1). The whole circuit can be integrated (highest CMRR) except R_1: the resistor controlling the gain.



Instrumentation amplifier gain

hypothesis: ideal op-amps

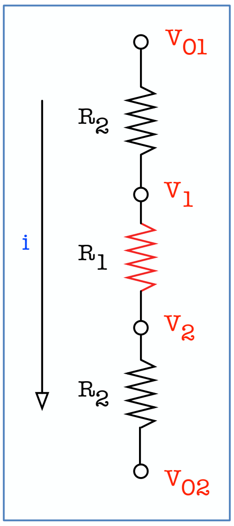


- The differential stage amplifies the difference between v_{01} and v_{02}.

$$v_{out} = \frac{R_4}{R_3} \cdot (v_{01} - v_{02})$$

- The input stage is the part where the relationship between the resistor R_1 and the gain is established.

analysis of the input stage:



$$i = \frac{v_1 - v_2}{R_1}$$

$$\frac{v_{o1} - v_1}{R_2} = i = \frac{v_1 - v_2}{R_1}\,;\,\frac{v_2 - v_{o2}}{R_2} = i = \frac{v_1 - v_2}{R_1}$$

$$v_{o1} = \frac{R_2}{R_1}(v_1 - v_2) + v_1$$

$$v_{o2} = -\frac{R_2}{R_1}(v_1 - v_2) + v_2$$

$$v_{o1} - v_{o2} = \frac{R_2}{R_1}(v_1 - v_2) + v_1 + \frac{R_2}{R_1}(v_1 - v_2) - v_2 =$$

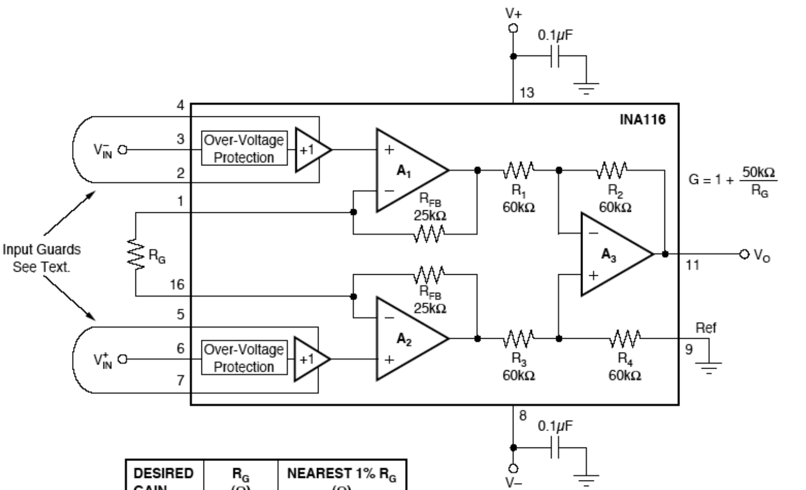
$$= 2\frac{R_2}{R_1}(v_1 - v_2) + (v_1 - v_2) = \left(1 + 2\frac{R_2}{R_1}\right) \cdot (v_1 - v_2)$$

$$v_{out} = \frac{R_4}{R_3} \cdot \left(1 + 2\frac{R_2}{R_1}\right) \cdot (v_1 - v_2)$$

Usually, $R_4=R_3$, then when $R_1=\infty$ G=1.
On the other hand, the case $R_1=0$ is not possible.

Instrumentation amplifier INA116

BURR - BROWN®

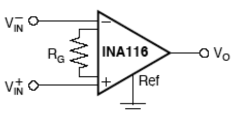


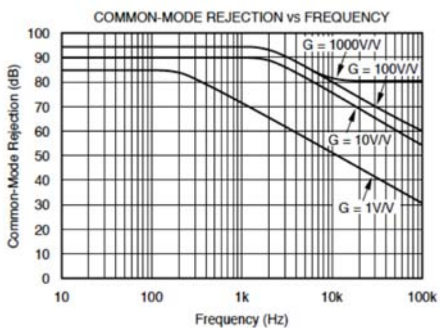
	INA116
Gain (V/V)	1 to 1000
Non-Linearity (+/-) (Max) (%)	0.005
Input Bias Current (+/-) (Max) (nA)	0.000025
Output Offset (+/-) (Max) (uV)	2000/G
Input Offset Drift (+/-) (Max) (uV/Degrees Celsius)	5
CMRR (Min) (dB)	86
Bandwidth at G=100 (Min) (kHz)	70
Noise at 1kHz (Typ) (nV/rt(Hz))	28
Vs (Min) (V)	9
Vs (Max) (V)	36

DESIRED GAIN	R_G (Ω)	NEAREST 1% R_G (Ω)
1	NC	NC
2	50.00k	49.9k
5	12.50k	12.4k
10	5.556k	5.62k
20	2.632k	2.61k
50	1.02k	1.02k
100	505.1	511
200	251.3	249
500	100.2	100
1000	50.05	49.9
2000	25.01	24.9
5000	10.00	10
10000	5.001	4.99

NC: No Connection.

Also drawn in simplified form:





Temperature dependence of the PN diode

- The characteristics of all junction devices depend on the temperature.
- PN silicon diodes can be used as temperature sensors. The characteristics of the sensor can be calculated from the fundamental equations of the device.

I/V function $$I = I_O \cdot \left[\exp\left(\frac{qV}{kT}\right) - 1\right]$$

Inverse current $$I_O = A \cdot q \cdot \left[\frac{D_p}{L_p N_d} - \frac{D_n}{L_n N_n}\right] \cdot n_i^2$$

$$n_i^2 = n_i \cdot p_i = N_C \cdot N_V \cdot \exp\left(-\frac{E_G}{kT}\right)$$

$$N_C = 2 \cdot \left(\frac{2\pi kT}{h^2}\right)^{\frac{3}{2}} \left(m_n^*\right)^{\frac{3}{2}}; \quad N_V = 2 \cdot \left(\frac{2\pi kT}{h^2}\right)^{\frac{3}{2}} \left(m_p^*\right)^{\frac{3}{2}}$$

$$n_i^2 = 4 \cdot \left(\frac{2\pi kT}{h^2}\right)^3 \left(m_n^* \cdot m_p^*\right)^{\frac{3}{2}} \exp\left(-\frac{E_G}{kT}\right)$$

$$L = \sqrt{D \cdot \tau}; \quad D = \mu \frac{kT}{q} \approx T^{-2} T \approx T^{-1}$$

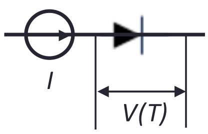
$$\tau = \frac{1}{v_{th} N_T \sigma} \approx T^{-\frac{1}{2}}$$

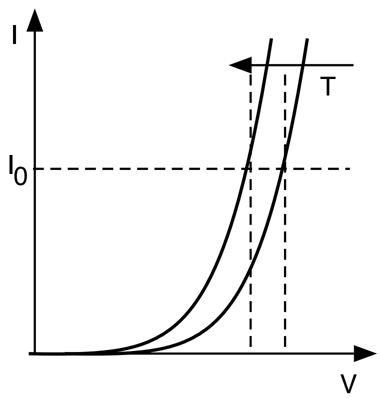
$$L \approx \sqrt{T^{-1} \cdot T^{-\frac{1}{2}}} \approx T^{-\frac{3}{4}}; \quad \frac{D}{L} \approx T^{-1} \cdot T^{\frac{3}{4}} \approx T^{-\frac{1}{4}}$$

$$I(T) = G \cdot T^3 \cdot \exp\left(\frac{qV - E_G}{kT}\right)$$

Temperature sensitivity of silicon diodes

Let us consider a diode biased with a constant current source





$$I(T) = G \cdot T^3 \cdot \exp\left(\frac{qV - E_G}{kT}\right) \Rightarrow V_D = \Phi + \frac{T}{\alpha} ln\left(\frac{I_1}{GT^3}\right)$$

Silicon: E_g=1.12 eV $\Phi = \frac{E_g}{q} = 1.12\ V;\ \alpha = \frac{q}{k} = 11600\ V/K$

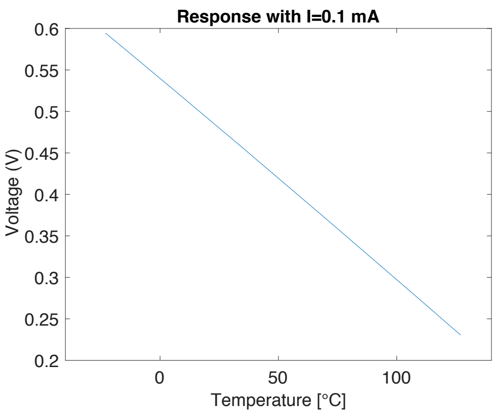
$$S = \frac{dV_D}{dT} = \frac{1}{\alpha} \cdot ln\left(\frac{I_D}{GT^3}\right) - \frac{3}{\alpha}$$

G=0.25 A/K³
biased with I_D=0.1 mA

S(-23°C)=-2.4 mV/K

S(127°C)=-2.5 mV/K

in practice is linear!



$$V(T) = V(T_0) - S \cdot (T - T_0)$$

equivalent to a thermistor:

$$R(T) = \frac{V_D(T)}{I_0} = R(T_0) - \frac{S}{I_0} \cdot (T - T_0) = R(T_0) \cdot [1 - \alpha(T - T_0)]$$

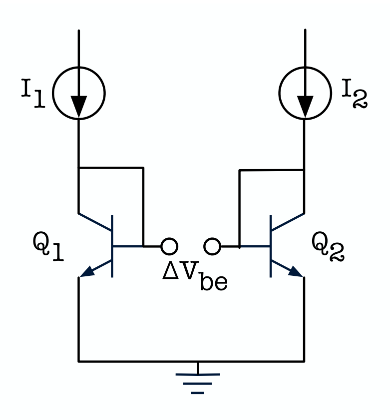
$$T = 0°C \rightarrow V_D(T_0) = 0.55\ V; R(T_0) = \frac{V_D(T_0)}{I_0} = 5.5\ K\Omega;$$

$$\alpha = \frac{S}{I_0 R(T_0)} = \frac{S}{V_D(T_0)} = \frac{2.4 \cdot 10^{-3}}{0.55} = 0.004\ K^{-1}$$

Comparable to a RTD

Signals Proportional to Absolute Temperature (PTAT)

- The dependence from the temperature is usually considered as a negative characteristics of the devices, and circuits are designed to compensate for such dependence.
- Two devices with the same parameters, except the dimension, are said matched.
- Matched devices can be fabricated only in integrated circuits



Let us consider two currents I_1 and I_2 injected in two matched diodes (in figure BJT as a diode)

$$I = I_S \cdot exp\left(\frac{qV_{be}}{kT}\right)$$

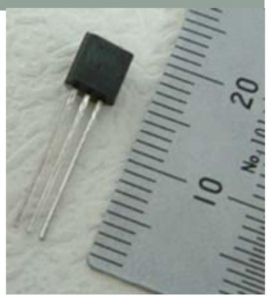
The voltage drop across the BJT bases is:

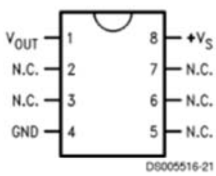
$$\Delta V = V_{be2} - V_{be1} = \frac{kT}{q}\left[ln\left(\frac{I_2}{I_S}\right) - ln\left(\frac{I_1}{I_S}\right)\right] = \left[\frac{k}{q} ln\left(\frac{I_2}{I_1}\right)\right]T$$

if the ratio between the currents in stable in temperature, ΔV is a PTAT signal.

The quantity k/q is: $\frac{k}{q} = \frac{1.38 \cdot 10^{-23} J \cdot K^{-1}}{1.6 \cdot 10^{-19} C} = 86 \frac{\mu V}{K}$

Integrated temperature sensor: LM35





- 3 terminals integrated circuit that acts as a voltage source proportional to the absolute temperature

  V=k T[C] k=10 mV/C
- Operating range: -55°C +150°C
- Working principle: Q_1=Q_2

Op amp action $\Rightarrow V_1 = V_2$

$$V_o - R_1 \cdot I_{c1} = V_o - R_2 \cdot I_{c2} \Rightarrow \frac{I_{c1}}{I_{c2}} = \frac{R_2}{R_1}$$

voltage drop across R_5 *is*:

$$V_{be1} - V_{be2} = V_T \cdot \ln\left(\frac{I_{c1}}{I_{s1}}\right) - V_T \cdot \ln\left(\frac{I_{c2}}{I_{s2}}\right) = V_T \cdot \ln\left(\frac{I_{c1}}{I_{c2}}\frac{I_{s2}}{I_{s1}}\right) = V_T \cdot \ln\left(\frac{R_2}{R_1}\right)$$

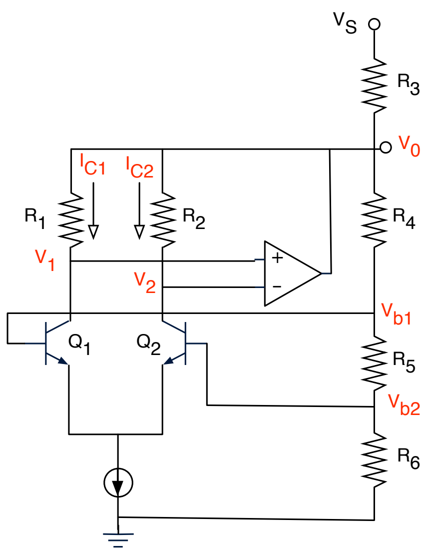
the current in R_5 is:

$$I = \frac{V_{be1} - V_{be2}}{R_5} = \frac{V_T}{R_5} \cdot \ln\left(\frac{R_2}{R_1}\right)$$

disregarding the base currents:

$$V_0 = (R_4 + R_5 + R_6) \cdot I = \frac{R_4 + R_5 + R_6}{R_5} V_T \cdot \ln\left(\frac{R_2}{R_1}\right)$$

$$V_0 = \left[\frac{R_4 + R_5 + R_6}{R_5} \cdot \frac{k}{q} \cdot \ln\left(\frac{R_2}{R_1}\right)\right] \cdot T = 10 \frac{mV}{K} \cdot T$$

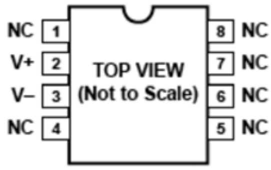


Integrated temperature sensor: AD590

Working principle

- Two terminals device that behaves as a current source whose current is proportional to the absolute temperature.

$$I = \mu \cdot T \quad ; \quad \mu = 1 \frac{\mu A}{K}$$

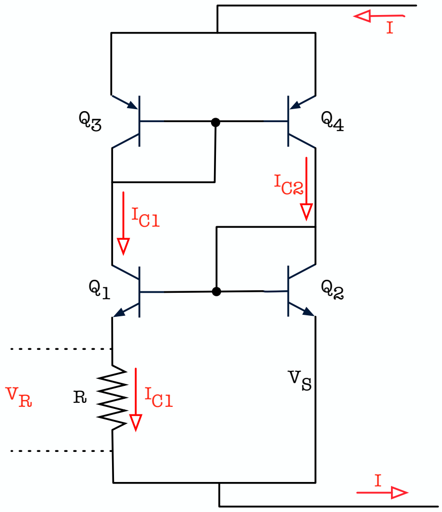


- range: from -55° C to +150° C
- $Q_3=Q_4$
- area Q_1=8*area Q_2

$$I_{c1} = I_{c2} = \frac{I}{2} \quad ; \quad \frac{I_{s1}}{I_{s2}} = 8$$

$$V_R = V_{BE1} - V_{BE2} = V_T \cdot ln\left(\frac{I_{c1}}{I_{s1}}\right) - V_T \cdot ln\left(\frac{I_{c2}}{I_{s2}}\right) = V_T \cdot ln\left(\frac{I_{c1}}{I_{c2}}\frac{I_{s1}}{I_{s2}}\right) = V_T \cdot ln(8)$$

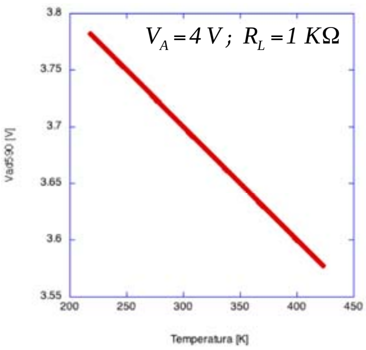
$$I = 2 \cdot I_{c2} = 2 \cdot \frac{V_R}{R} = 2 \cdot \frac{V_T}{R} \cdot ln(8) = 2 \cdot \frac{k \cdot T}{q} \cdot \frac{1}{R} \cdot ln(8) = \left[2 \cdot \frac{k}{q} \cdot \frac{1}{R} \cdot ln(8)\right] \cdot T$$



R is chosen in order to achieve μ=1 μA/K

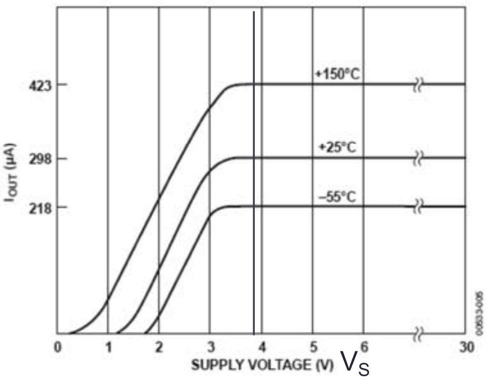
Circuits with AD590

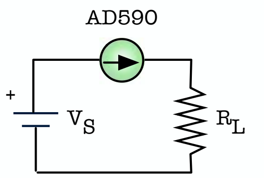
$$V_{AD590} = V_A - R_L \cdot \mu \cdot T$$



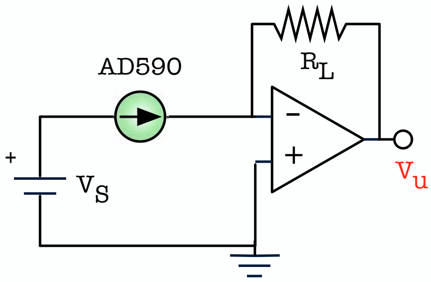
I/V curve:

The supply voltage has to exceed 4 V in order to activate the sensor.



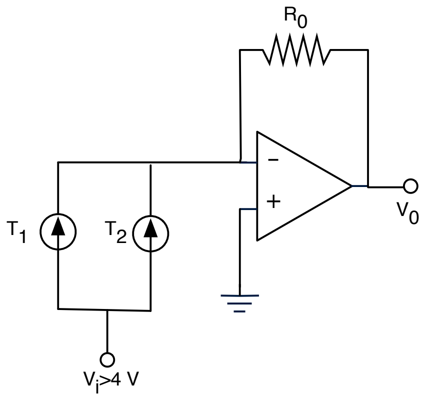


V_S>4 V $\quad V_u = R_L \cdot \mu \cdot T$



$$V_u = -R_L \cdot \mu \cdot T$$

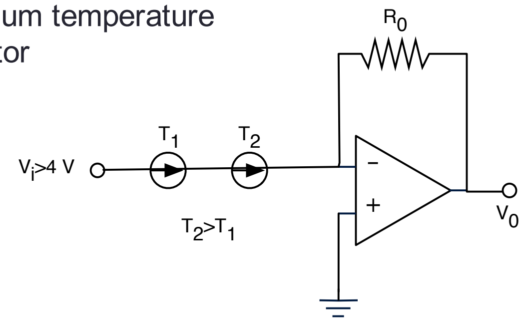
Parallel and series connections

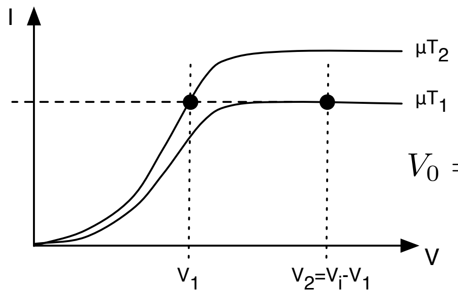


parallel connection:
equivalent to a virtual sensor applied to the mean temperature.
The total sensitivity is n times the sensitivity of the single sensor.

$$I_1 = \mu T_1$$
$$I_2 = \mu T_2$$
$$V_0 = -R_0 \cdot (I_1 + I_2) = -R_0 \cdot (\mu T_1 + \mu T_2) = -\mu R_0 \cdot (T_1 + T_2) = -2\mu R_0 \cdot \frac{T_1 + T_2}{2}$$
$$\rightarrow V_0 = -R_0 \cdot n\mu \cdot \langle T \rangle$$

series connection:
minimum temperature detector





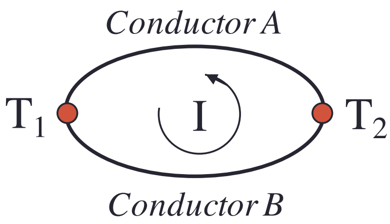
$$V_0 = -R_0 \cdot \mu T_1$$

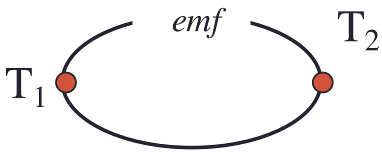
Thermoelectricity

- Effects of the temperature on the electric circuits:
  - All the parameters of materials depend on temperature
    - Fermi level, Band gap, density of states……
  - Then also the built-in potentials depend on temperature
  - Difference of temperature elicits a diffusion current also in a metal
- Thermoelectric effects are manifested if the temperature in a circuit is not uniform
  - Uniform temperature is an implicit assumption in network theory.
- Three main thermoelectric effects:
  - Seebeck effect
  - Peltier effect
  - Thomson effect

Seebeck effect

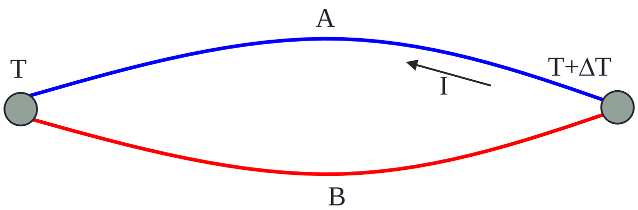
- Seebeck experiment (1821): a current flows in a closed circuit formed by two different conductors when the two junctions are held at different temperature.
- If the network is opened, at any point, a voltage drop is observed.
- The couple of conductors is called *thermocouple*. The quantities of current and voltage are proportional to the difference of temperature between the junctions.
- If one of the junctions is held at constant temperature the other junction can be used to measure an unknown temperature. The fixed temperature is called reference temperature.
- The sensitivity of the thermocouple is called *thermoelectric power*. It is a function of the temperature.
- The properties of the thermocouple does not depend on the dimension of the conductors. The same materials provide the same sensitivity disregarding the size.

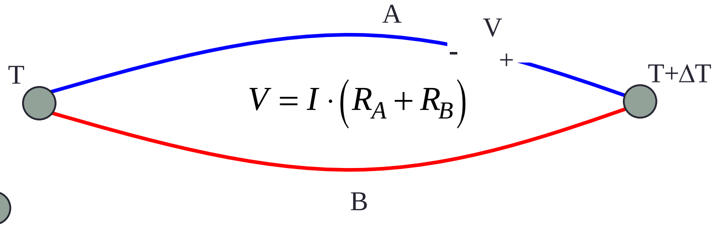


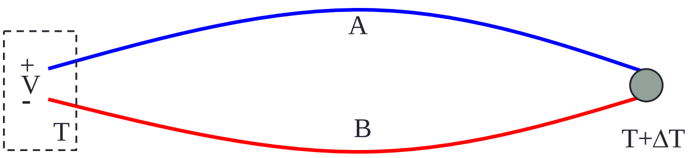


More on Seebeck effect

- The Seebeck effect can be explained as the sum of two diffusion currents due not to a gradient of concentration but a gradient of temperature. If the materials are different the currents are different and the total current is different than zero.
- The open circuit voltage correspond to the emf necessary to obtain the closed circuit current
- The direction of the current depends on the properties of the materials: Considering the figure below, the conductor A is said positive respect to B when the current flows from A to B at the cold junction.
- The voltage drop can be measured opening the thermocouple at any point, included the junction itself. It is not necessary a physical junction, but the two elements have to be kept at the same temperature.





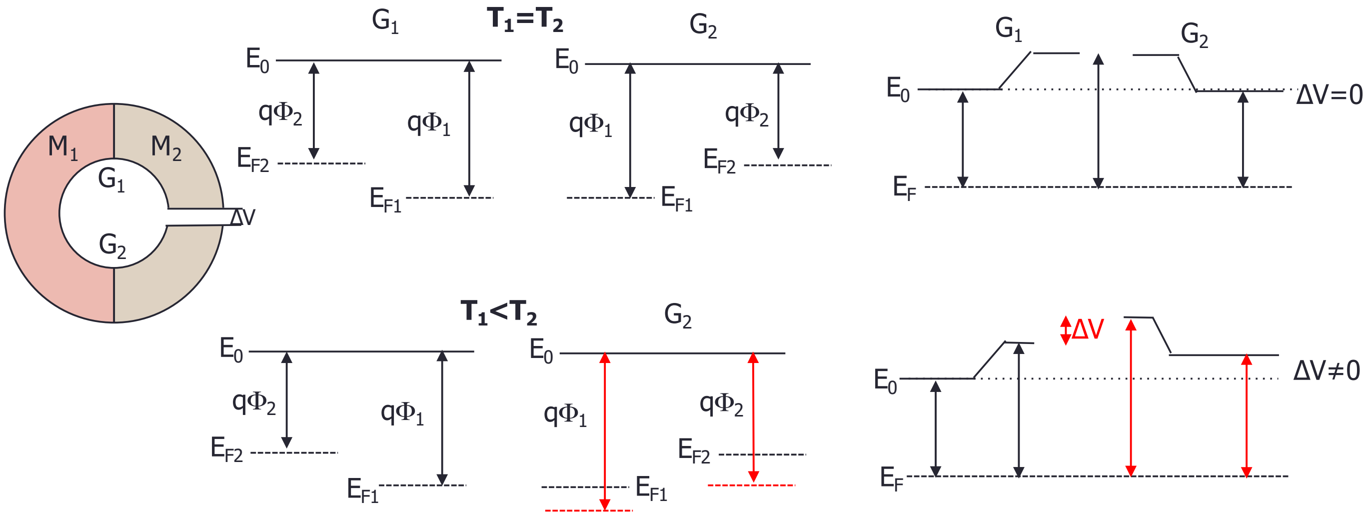


Seebeck effect and Fermi level

- In semiconductors, the Seebeck effect can be explained with dependence of the Fermi level with the temperature.
- The Seebeck effect can be observed only if the Fermi levels of the conductors are different.

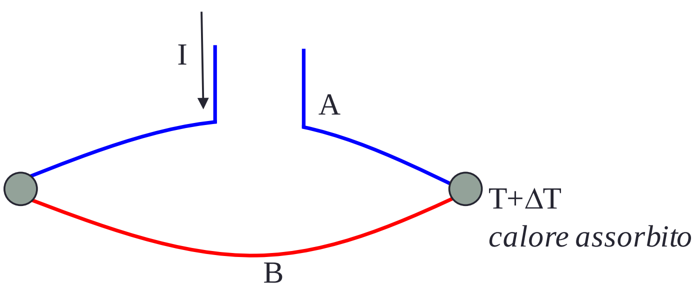
$$E_F = E_{F0}\cdot\left[1-\frac{\pi^2}{12}\cdot\left(\frac{kT}{E_{F0}}\right)^2-\frac{\pi^4}{80}\cdot\left(\frac{kT}{E_{F0}}\right)^4+\dots\right]$$

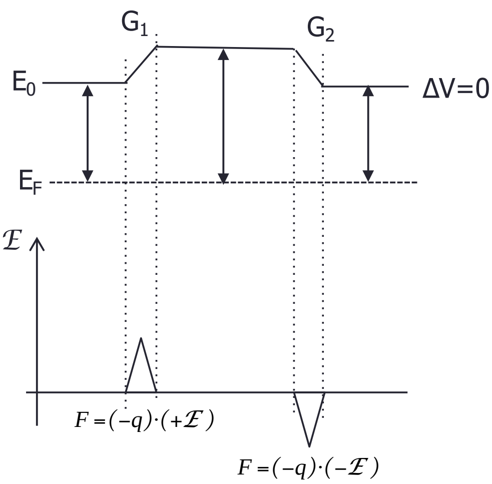
E_{F0}: Fermi level at 0 K



Peltier effect

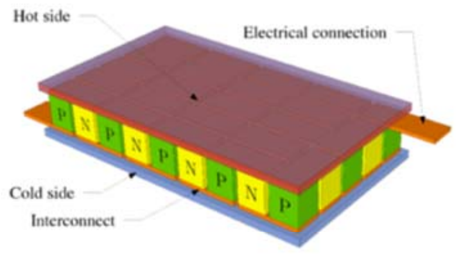
- When a current flows in a circuit formed by two different conductors, heat is released and absorbed at the two junctions.
  - The built-in potentials at the two junctions have the same magnitudes but different signs. The electrons crossing the junctions are either accelerated or decelerated by the built-in potential. In order to maintain constant the current they acquire from the lattice, in case of the decelerating field, an extra kinetic energy, and in the case of a accelerating field they release the extra kinetic energy. As a consequence the material is cooled at one junction and heated at the other.
- Peltier effect is used for thermoelectric refrigeration and heating.





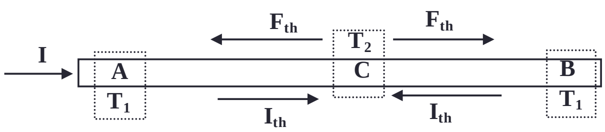
Peltier cells

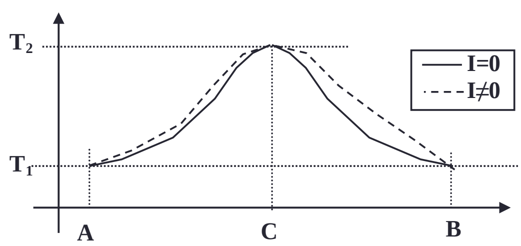
- Peltier effect requires not only a built-in potential (found in any junction) but also a sufficiently long depletion layer (found only in semiconductors). For this reason, thermoelectric heating/cooling is done with series of semiconductors junctions.



Thomson effect

- *The Thomson effect is an asymmetry in the temperature distribution observed when a current flows in a conductor where temperature gradients are permanently maintained.*
- Let us consider a constant current injected in a homogeneous conductor. The extremities of the conductor are kept at temperature T_1 and the central part is held at temperature T_2 with $T_2>T_1$.
- Due to Joule effect, the temperature of the conductor tends to increase, but the temperature at the extremities and at the center are kept fixed.
- Two opposite gradients of temperature are present in the conductor, this means that there are two diffusion currents from the center to the extremities. The injected current then is alternatively summed and subtracted to the diffusion current. Since the total current has to be constant (Kirchhoff law) the electrons have to accelerate when the diffusion current is opposite and to decelerate when the diffusion current is in the same direction with respect to the imposed current. The energy for acceleration and deceleration is provided by the lattice vibration (heat).
- Then Joule effect appears less efficient when the diffusion current is opposite and more efficient when the diffusion current is in the same direction of the external current.





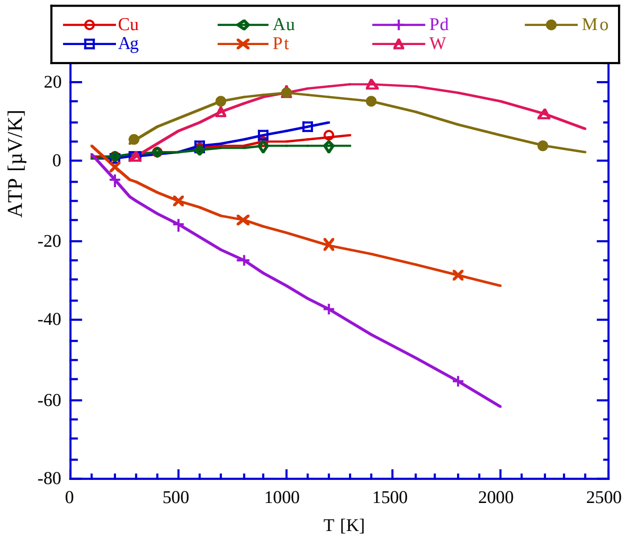
Absolute Thermoelectric Power (ATP)

a.k.a. Seebeck coefficent

- It can be demonstrated that, given a couple of conductors, the termoelectric power of a thermocouple is the difference of the absolute thermoelectric powers (ATP)

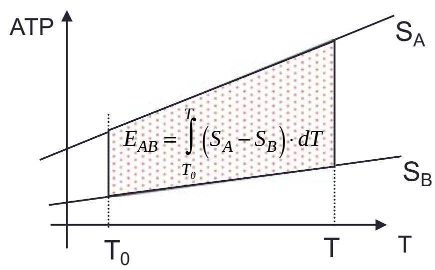
$$TP = \frac{dV_{AB}}{dT} = ATP_A - ATP_B$$

- The ATP is a function of the temperature, then also the TP of the thermocouple is a function of the temperature and then the curve of response is not linear.



Criteria for linearity

- Let us consider two thermo-elements whose ATP changes linearly with the temperature.



$$S_A = c_1 + m_A \cdot T \qquad S_B = c_2 + m_B \cdot T \;\Rightarrow\; \frac{dV_{AB}}{dT} = c_3 + (m_A - m_B) \cdot T\,;\; c_3 = c_1 - c_2$$

- The emf generated by the thermocouple is the area between the two ATP curves calculated from the temperature of the two junctions.

$$V_{AB} = \int_{T_0}^{T} \frac{dE_{AB}}{dT} \cdot dT = \int_{T_0}^{T} c_3 + (m_A - m_B) \cdot T \cdot dT = c_3 \cdot (T - T_0) + \frac{1}{2}(m_A - m_B) \cdot (T^2 - T_0^2)$$

- The non linearity disappears, and the TP is constant, if the ATP of the two elements are parallel with respect to temperature ($m_A = m_B$).

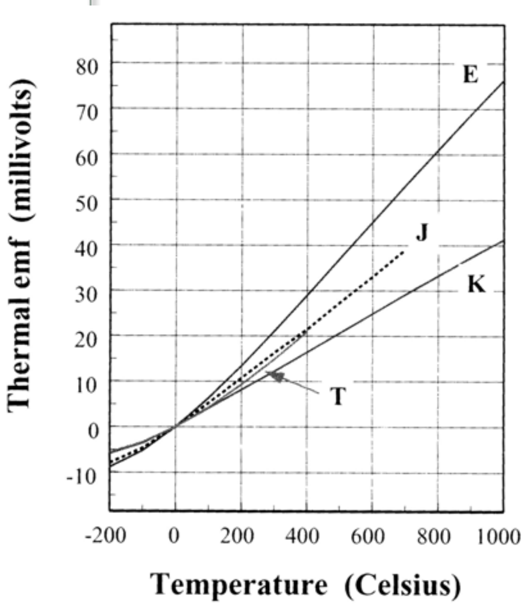
$$V_{AB} = \int_{T_0}^{T} c_3 \cdot dT = E_0 + c_3 \cdot (T - T_0) \;\Rightarrow\; TP = \frac{dV_{AB}}{dT} = c_3$$

Thermocouples standards

- Special alloys have been developed to maximize the sensitivity and the linearity over a large range. Their combination defines thermocouple standards.

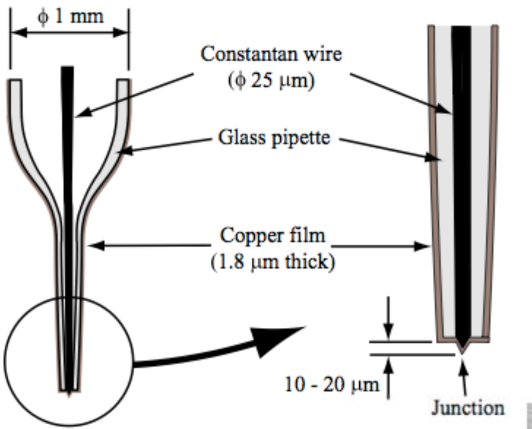
Type	Metal A - Metal B	Temperature Range (°C)	Sensitivity (μV/K)
Type E	Chromel - Constantan	-200 to +900	75
Type J	Iron - Constantan	0 to +750	50
Type K	Cromel - Alumel	-200 to +1250	42
Type T	Copper - Constantan	-200 to +350	50

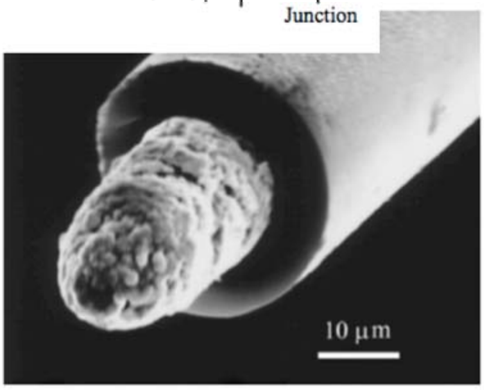
Chromel: 90% Ni, 10% Cr
Constantan: 55% Cu, 45% Ni
Alumel: 95% Ni, 2% Mn, 2% Al, 1% Si

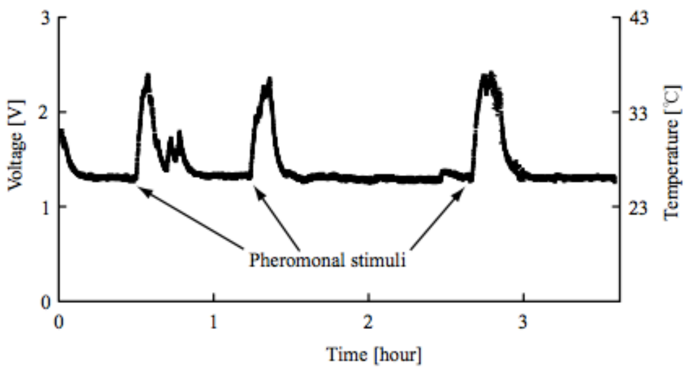


Micro thermocouple for biological measurements

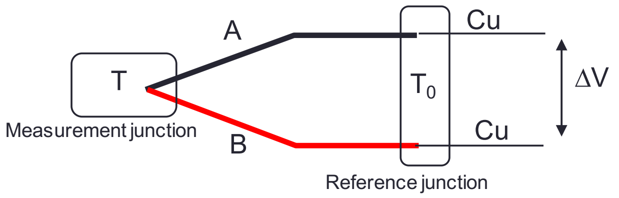
To verify its utility, the microthermocouple probe was inserted into the dorsal longitudinal muscle (DLM) of a male silkworm moth. In response to the sex-attractant pheromone of a female moth, a male begins to beat its wings using the DLM, which causes the temperature of the DLM to rise. The DLM temperature was monitored over 3 h, and the temperature was seen to rise rapidly by 10 – 11 °C after pheromonal stimulation, thereafter return to the original temperature at rest after 15 – 20 min, as shown in Fig. 3.



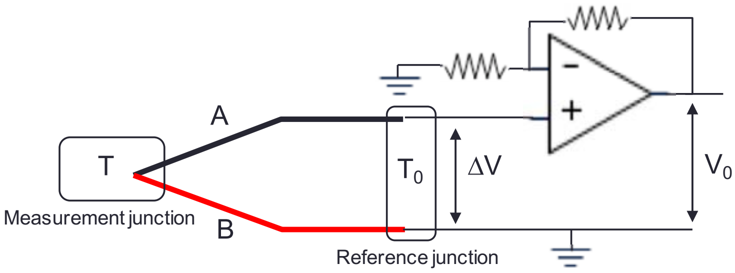




Measurement setup



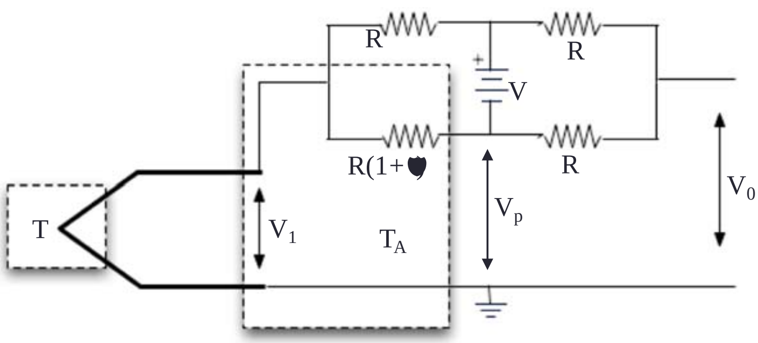
General measurement setup



Potentiometric measurement
The magnitude of the emf is of the order of hundreds of μV then it is necessary to amplify the signals.
To ensure the potentiometric measurement the input impedance of the amplifier has to be very large (instrumentation amplifier).

Compensation of the reference temperature

- An RTD can be used to make a signal that is independent from the reference temperature (T_A).
- The RTD is connected to a bridge in series with the thermocouple. The output voltage is measured through a high impedance amplifier (no current in the thermocouple)



$$R_{RTD} = R\cdot(1+\delta) = R\cdot(1+\alpha\cdot(T_A - T_0))$$

$$V_i = -TP\cdot(T-T_A);$$

$$V_0 - V_i = \frac{V}{2} - V\cdot\frac{R(1+\delta)}{R+R(1+\delta)} = \frac{V}{2} - V\cdot\frac{1+\delta}{2+\delta} = -\frac{\delta\cdot V}{2\cdot(2+\delta)}$$

$$V_0 = -TP\cdot(T-T_A) - \frac{\alpha\cdot(T_A-T_0)\cdot V}{4+2\cdot\alpha\cdot(T_A-T_0)} = -TP\cdot(T-T_0) + TP\cdot(T_A-T_0) - \frac{\alpha\cdot(T_A-T_0)\cdot V}{4+2\cdot\alpha\cdot(T_A-T_0)}$$

If δ<< 1 $\cong -TP\cdot(T-T_0) + TP\cdot(T_A-T_0) - \frac{\alpha\cdot(T_A-T_0)\cdot V}{4}$

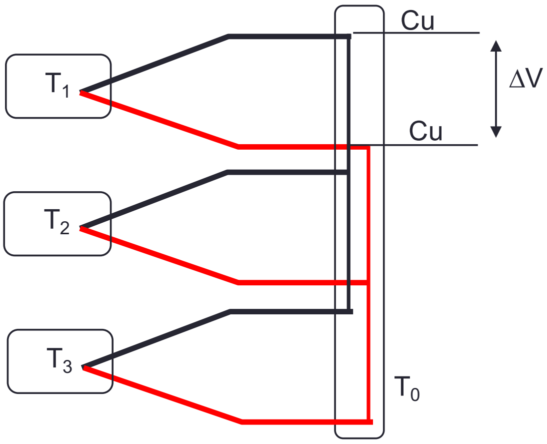
$$\text{independence from reference}: -TP\cdot(T_A-T_0) = \frac{\alpha\cdot(T_A-T_0)\cdot V}{4} \Rightarrow TP = \alpha\cdot\frac{V}{4}$$

RTD platinum α=0.0039 1/K
Type K TP=50 μV/K
-> V=51 mV

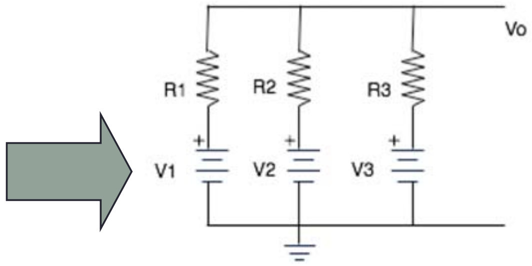
Thermocouples: parallel connection

measure of average temperature

- Identical thermocouples with the same R



Equivalent circuit



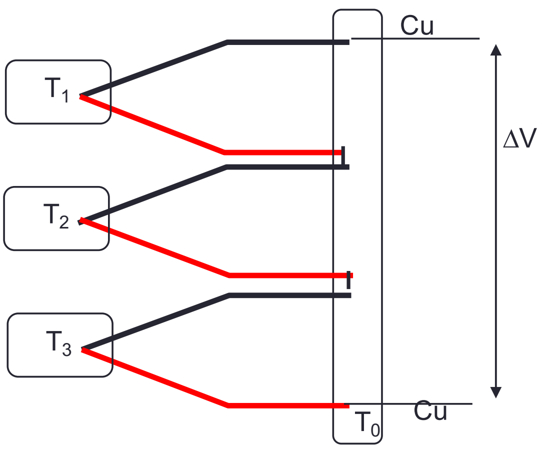
If the voltage is measured through a high impedance amplifier, the total current at the node is zero.

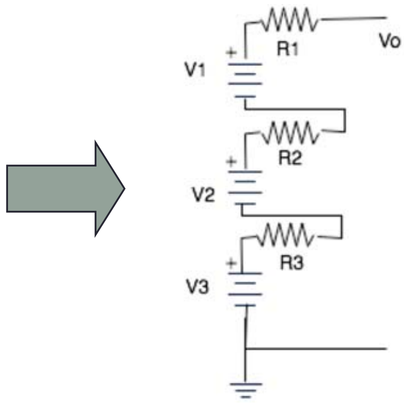
$$\sum \frac{V_i - V_o}{R} = 0 \longrightarrow V_o = \frac{\sum_i \frac{V_i}{R}}{\sum_i \frac{1}{R}} = \frac{1}{n} \cdot \sum_i V_i$$

The output voltage is the average voltages of the thermocouples. Then it corresponds to one thermocouple applied to the average temperature location.

Thermocouples: series connection

measure of average temperature





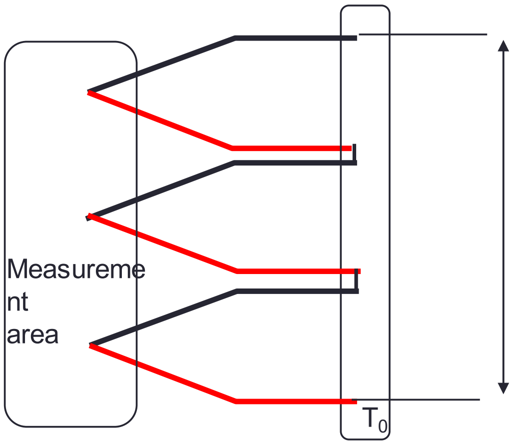
equivalent circuit

$$V_0 = \Delta V_1 + \Delta V_2 + \Delta V_3 = \alpha \cdot (T_1 - T_0) + \alpha \cdot (T_2 - T_0) + \alpha \cdot (T_3 - T_0) = \alpha \cdot (T_1 + T_2 + T_3 - 3 \cdot T_0) =$$

$$= 3 \cdot \alpha \cdot \left(\frac{T_1 + T_2 + T_3}{3} - \frac{3 \cdot T_0}{3} \right) = 3 \cdot \alpha \cdot (\overline{T} - T_0)$$

The output voltage corresponds to a thermocouple whose thermoelectric power is n times larger than the individual sensor and applied to the average temperature location.
Amplification of sensitivity: thermopile.

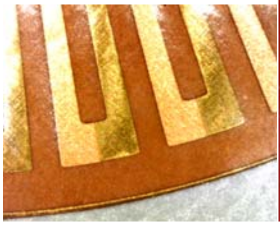
Thermopile

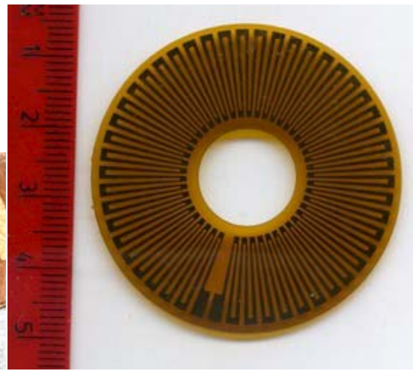


- Thermopiles are composed by n identical thermocouples connected in series.
- The output voltage is:

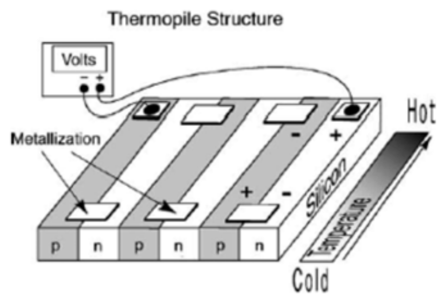
$$V_o = n \cdot TP_{AB} \cdot \Delta T$$

- The thermopile is a more efficient temperature sensor, but the measurement area is increased with respect to a single thermocouple.
- The total resistance increases, then the termopile is more affected by thermal noise.
  - This is important for semiconductor thermopiles
- Thermopiles can be fabricated in silicon and then integrated





Integrated thermopile

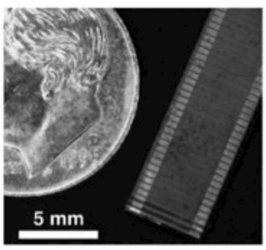


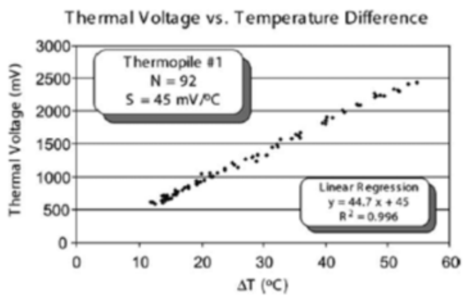
Semiconductors show the largest Seebeck coefficient, then semiconductors thermopiles are very sensitive. Integrated thermopiles can measure small temperature differences across small distances.

Table 1
Seebeck coefficients of metals relative to platinum

Material	Seebeck coefficient (μV/°C)
Antimony	+48.9
Chromel	+29.8
Tungsten	+11.2
Gold	+7.4
Copper	+7.6
Silver	+7.4
Aluminum	+4.2
p-Silicon, $\rho = 0.0035\ \Omega$ cm	+450
p-Germanium, $\rho = 0.0083\ \Omega$ cm	+420
Platinum	0.00
Calcium	−5.1
Alumel	−10.85
Cobalt	−13.3
Nickel	−14.5
Constantan	−37.25
Bismuth	−73.4
n-Silicon, $\rho = 0.0035\ \Omega$ cm	−450
n-Germanium, $\rho = 0.69\ \Omega$ cm	−548

Measurements were made with the reference junction at 0 °C and the hot junction at 100 °C [25].

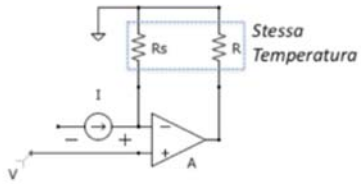




Thermal feedback

11
Nel circuito seguente la resistenza R_S è un sensore RTD. Il sensore è accoppiato termicamente ad una resistenza R. La resistenza R scambia calore con l'ambiente circostante (che ha temperatura fissa T_A) attraverso una costante di dissipazione termica δ. Si consideri l'amplificatore operazionale ideale.

- **Calcolare il valore della tensione V che consente di portare la temperatura della resistenza R a T=70°C.**



$$R_S = R_0 \cdot [1+\alpha \cdot (T-T_0)]$$

$$R_0 = 100\Omega;\ T_0 = 20°C;\ \alpha = 0.005K^{-1}$$

$$I = 10mA;\ R = 300\Omega;$$

$$\delta = 10^{-3}\frac{W}{K}$$

$$V = R_S \cdot I = R_0 \cdot [1+\alpha \cdot (T-T_0)] \cdot I$$

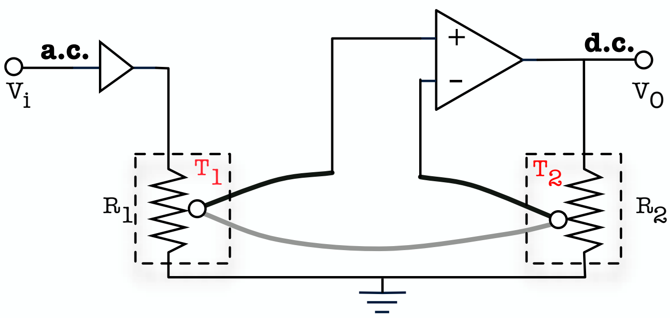
$$V = 100 \cdot [1+0.005 \cdot (70-20)] \cdot 0.01 = 1.25V$$

Analog rms voltmeter

- The rms value of a a.c. voltage signal is equivalent to a d.c. signal that dissipates the same amount of power.
- The above definition can be applied in a circuit that exploits the thermo-electric feedback of an operational amplifier.
- The signal v_i (a.c.) is applied to a resistor R_1 where it dissipates a power P_1 that increases the temperature of R_1. The thermocouple measures the difference of temperature between two identical resistors. The output of the thermocouple is applied to the input terminals of an op amp. The thermocouple acts a negative feedback network, then the op amp output voltage forces the voltage drop across the input to be zero. Then the thermocouple output is zero, and this happens if the temperatures of the resistors are equal, and since the resistors are identical the op amp ouput voltage v_0 (d.c.) is the rms of v_i.

$$v_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T v^2(t) \cdot dt}$$

$$v(t) = V \cdot sin\left(\frac{2\pi}{T}t\right) \Rightarrow v_{rms} = \frac{V}{\sqrt{2}}.$$



Integrated analog rms-dc converter

- Thermocouples can be replaced by a couple of matched temperature sensors such as diodes.
- An integrated device based on a couple of identical diode temperature sensor is commercially available.