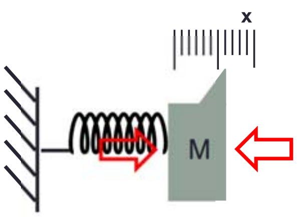
SENSORS FOR MECHANICAL QUANTITIES

subjects:

- Force sensors
    - strain gauges
- Accelerometers
- Gyroscope
- Pressure sensors
- Flow sensors
- Piezoelectric sensors

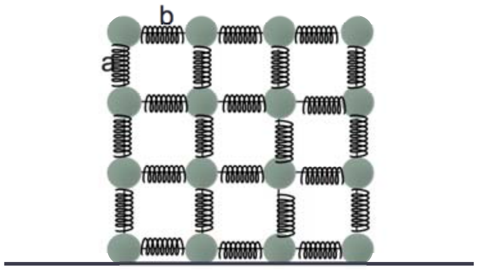
Force sensors (load cells)

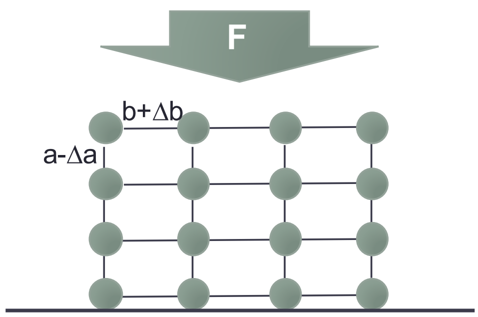
- Force sensors are made by an elastic probe at which the force is applied. The equilibrium between elastic (internal) force and the applied force changes the dimensions of the elastic probe
- Ideally, the elastic probe is a mass-less spring:



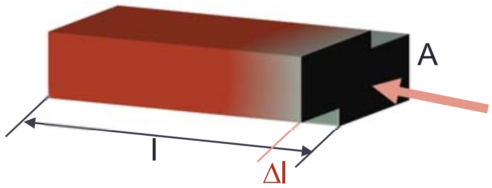
$$Fappl = F_e = K \cdot (x - x_{eq})$$

- In practice the elastic probes are solid bodies (bonds form a network of springs). In the elastic regime (when the applied force does not permanently deform the body) the Hooke law holds and the magnitude of the deformation is proportional to the applied force.





Deformation of solid bodies



- **stress** (σ): applied force divided by the area of application.

$$\sigma = \frac{F}{Area}\left[\frac{N}{m^2}\right] = [Pa]$$

- As a consequence of the applied stress the body is deformed. The magnitude of body deformation along a specified direction is called **strain** (ε). It is a dimensionless quantity

$$\varepsilon = \frac{\Delta l}{l}$$

- In the elastic regime, stress and strain are linearly proportional according to the Hooke's law.

$$\sigma = E \cdot \varepsilon$$

- E is the Young's module. It is a measure of the elasticity of the body.

Poisson's ratio

- The deformation along one direction gives rise to a deformation, of opposite sign, along the two orthogonal directions.

$$\varepsilon_x = \frac{\Delta x}{x} \Rightarrow \begin{cases} \varepsilon_y = \dfrac{\Delta y}{y} = -\nu \cdot \varepsilon_x \\ \varepsilon_z = \dfrac{\Delta z}{z} = -\nu \cdot \varepsilon_x \end{cases}$$

- The Poisson's ratio (ν) measures the relationship between the strain in the applied direction and the strains in the orthogonal directions.
- ν is between 0 and 0.5
  - if ν=0.5 the volume remains constant: incompressible solid

Resistive strain gauges

- Sensors that measure the deformation of solid bodies are called strain gauges.
- Strain gauges can be based on several physical principles (e.g. optical interferometry) but the most simple and diffused are the resistive strain gauges.
- These are resistors fixed onto the body surface so that the deformation of the body is applied to the resistor material.
- The gauges can be soldered, glued, or screwed. The quality of the connection is crucial for a reliable measurement.

Sensitivity of resistive strain gauges

- Resistive strain gauges exploit the relationship between the electric resistance and the dimensions of the conductor. Since, stretches and compressions change the distance between the atoms, also the resistivity (mobility and density of charge carriers) is affected by the strain.
- The resistance of a conductor of resistivity ρ, length l, and cross-section A is:

$$R = \rho \cdot \frac{l}{A}$$

- Strain changes all terms: the total variation is:

$$R = \rho \cdot \frac{l}{A}$$

$$dR = \frac{\partial R}{\partial \rho} \cdot d\rho + \frac{\partial R}{\partial l} \cdot dl + \frac{\partial R}{\partial A} \cdot dA = \frac{l}{A} \cdot d\rho + \frac{\rho}{A} \cdot dl - \rho \cdot \frac{l}{A^2} \cdot dA$$

consider that:

$$\frac{l}{A} = \frac{R}{\rho}; \quad \frac{\rho}{A} = \frac{R}{l}; \quad \rho \cdot \frac{l}{A^2} = \frac{R}{A}$$

$$= \frac{R}{\rho} \cdot d\rho + \frac{R}{l} \cdot dl - \frac{R}{A} \cdot dA$$

$$\Rightarrow \boxed{\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{dA}{A}}$$

this is sometimes called logarithmic derivative $\ln R = \ln \rho + \ln l - \ln A \quad \Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{dA}{A}$

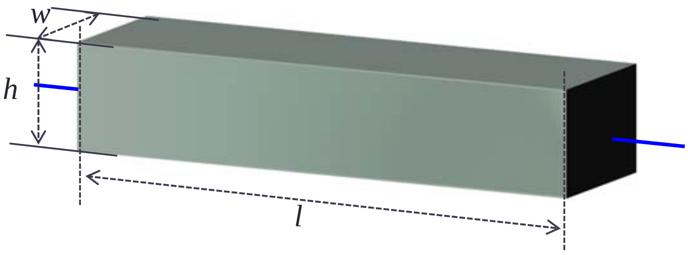
Sensitivity of resistive strain gauges

- Considering a force applied along the main direction of a resistor, we have:

$$\frac{\Delta l}{l} = \varepsilon$$

$$\frac{\Delta A}{A} = \frac{\Delta w}{w} + \frac{\Delta h}{h} = -\upsilon \cdot \varepsilon - \upsilon \cdot \varepsilon = -2 \cdot \upsilon \cdot \varepsilon$$

$$\frac{\Delta \rho}{\rho} = \chi \cdot \varepsilon$$



$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A} = \chi \cdot \varepsilon + \varepsilon - (-2 \cdot \upsilon \cdot \varepsilon) = (1 + 2 \cdot \upsilon + \chi) \cdot \varepsilon = K \cdot \varepsilon$$

- χ is the piezoresistivity. It takes into account the effects of the strain on the mobility and, for semiconductors, on the carrier charge density.
- ν is the Poisson's ratio of the material of the resistor.
- K is the **gauge factor**, it is the normalized sensitivity of the resistance to the strain.

Strain gauges: conductors and semiconductors

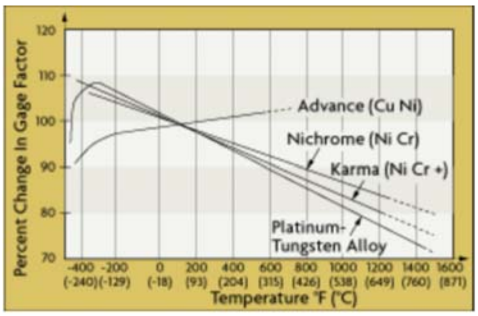
- Piezoresistivity in metals is almost negligible. Since Poisson's ratio is about 0.3, the gauge factor is of the order of few units.
- In semiconductors the piezoresistivity is dominant. The gauge factor is more than one order of magnitude larger than in metals but it is not linear with the strain. Typical values are:

$$N\text{-type:} \quad \frac{\delta R}{R} = 120\varepsilon + 4\varepsilon^2$$

$$P\text{-type:} \quad \frac{\delta R}{R} = -110\varepsilon + 10\varepsilon^2$$

Strain gauges are made of the similar materials used for thermistors, then the sensitivity to temperature is not negligible.

Semiconductor gauges cannot be arbitrarily shaped and they show a larger cross-interference from temperature. Largely used in integrated sensors.



The complete characteristics is:

$$R = R_0 \cdot (1 + k(T) \cdot \varepsilon(T) + \alpha \cdot T)$$

Three sources of sensitivity to the temperature

- α: temperature coefficient of the gauge material;
- k(T) dependency of the gauge factor (resistivity) from the temperature
- ε(T): dependency of the strain from the temperature

The connection in a Wheatstone bridge helps in reducing the influence of temperature

Strain gauges in a Wheatstone bridge

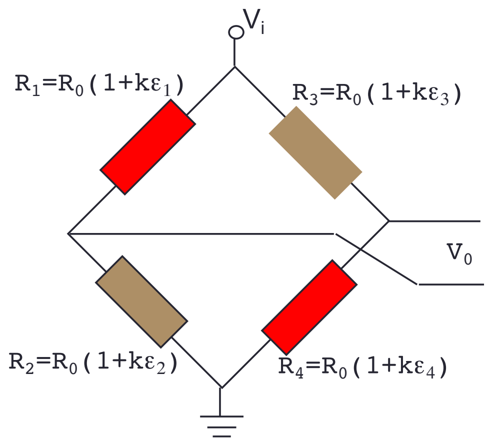
- Given 4 identical strain gauges undergoing four different strains and neglecting the temperature changes.

$$R_i = R + \Delta R_i = R \cdot (1 + k \cdot \varepsilon_i)$$

- If $k\varepsilon$ is small, the output of the bridge can be written as:

$$\frac{V_o}{V_i} = \frac{k}{4}(+\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4)$$

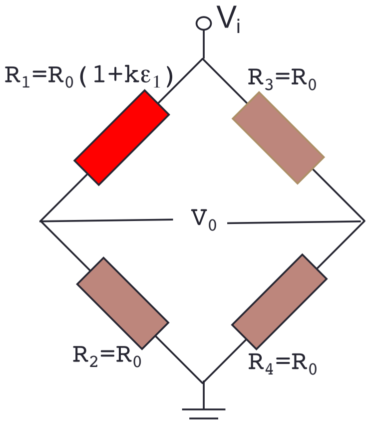
- The sign of each gauge can be used to increase the sensitivity and to reduce the interferences.



Quarter, half, and full bridge

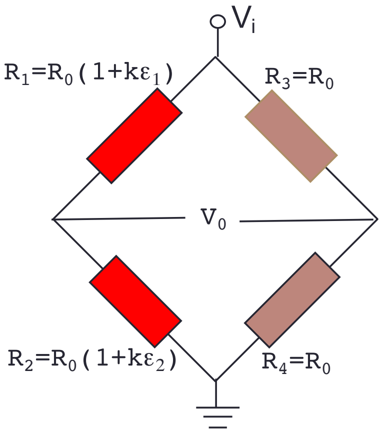
- The bridge sensitivity depends on the number of active gauges
- The resistance of the fixed elements are identical to the null strain gauge resistance.

Quarter bridge



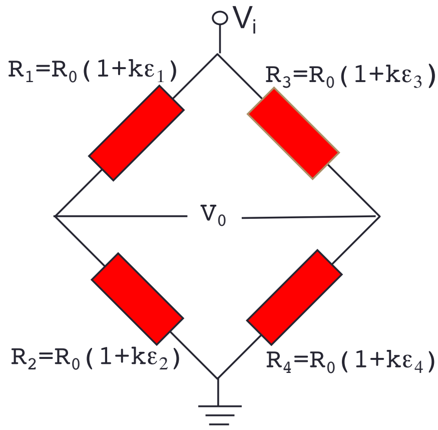
$$\frac{V_o}{V_i} = \frac{k}{4}(\varepsilon_1)$$

Half bridge



$$\frac{V_o}{V_i} = \frac{k}{4}(\varepsilon_1 - \varepsilon_2)$$

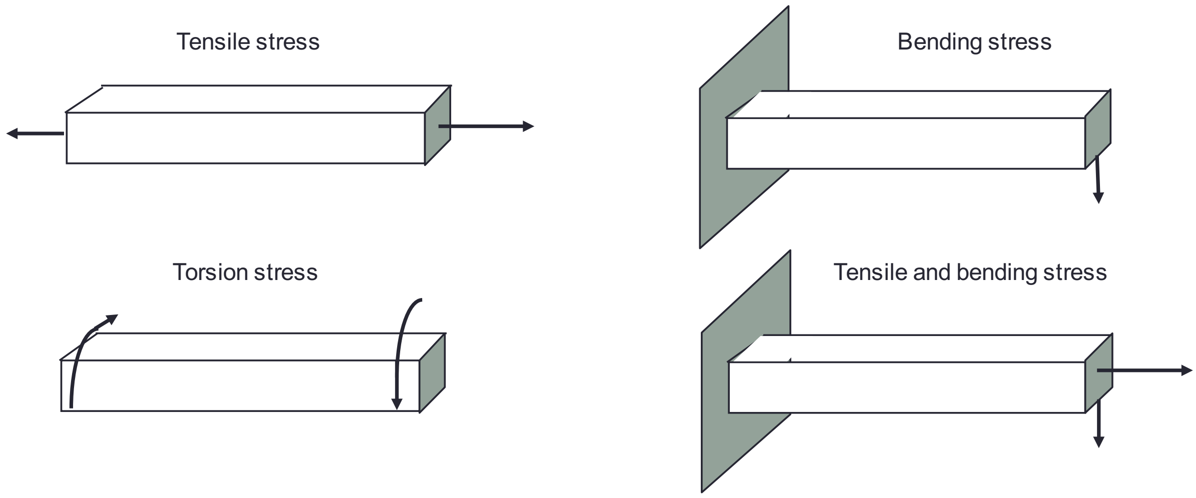
Full bridge



$$\frac{V_o}{V_i} = \frac{k}{4}(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4)$$

Measurement configurations

- The geometry of the arrangement and induced deformation (Poisson's ratio) allows for a non-trivial combination of more gauges at once.



Tensile stress in a rod

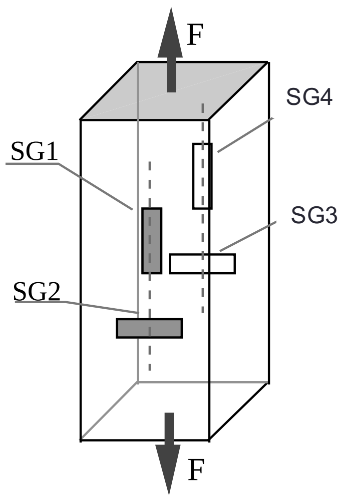
- The stress is applied along the main direction of the rod. The largest strain (ε) occurs along the direction of the force.
- Due to the Poisson's ratio strains of opposite sign occurs along the orthogonal directions.

$$\varepsilon_n = -\nu \cdot \varepsilon$$

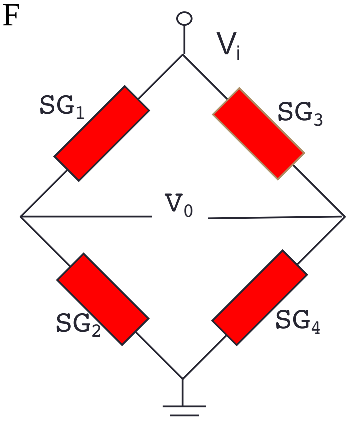
- The full bridge configuration can be obtained with two gauges oriented along the force and two gauges in the orthogonal direction.
- Poisson's ratio for solid bodies in elastic regime is about 0.3 then:

$$\frac{V_0}{V_i} = \frac{1}{4} \cdot \left[k \cdot \varepsilon - (-k \cdot \nu \cdot \varepsilon) - (-k \cdot \nu \cdot \varepsilon) + k \cdot \varepsilon \right] = \frac{k}{4} 2 \cdot (1+\nu) \cdot \varepsilon$$

$$\nu \cong 0.3 \Rightarrow \frac{V_0}{V_i} = \frac{k}{4} \cdot 2.6 \cdot \varepsilon$$

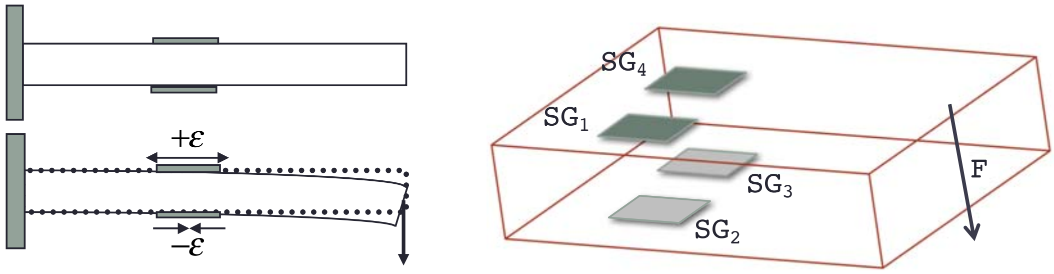


$$\delta_1 = k \cdot \varepsilon$$
$$\delta_4 = k \cdot \varepsilon$$
$$\delta_2 = -k \cdot \nu \cdot \varepsilon$$
$$\delta_3 = -k \cdot \nu \cdot \varepsilon$$



Bending of a plate or a rod

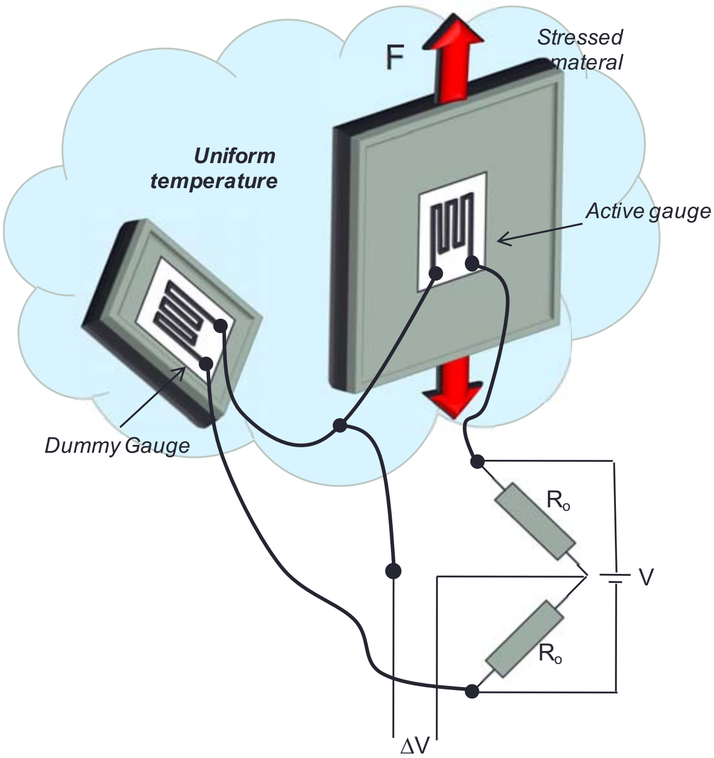
- Bending induces a stretch (+ε) on the upper surface and and a compression (-ε) on the lower surface. The median plane remains unperturbed
- The full bridge can be obtained placing two gauges on the upper surface and two gauges on the lower surface



$$\delta_1 = k \cdot \varepsilon$$
$$\delta_4 = k \cdot \varepsilon$$
$$\delta_2 = -k \cdot \varepsilon$$
$$\delta_3 = -k \cdot \varepsilon$$

$$\frac{V_0}{V_i} = \frac{1}{4} \cdot (\delta_1 - \delta_2 - \delta_3 + \delta_4) = \frac{1}{4} \cdot [k \cdot \varepsilon - (-k \cdot \varepsilon) - (-k \cdot \varepsilon) + k \cdot \varepsilon] = k \cdot \varepsilon$$

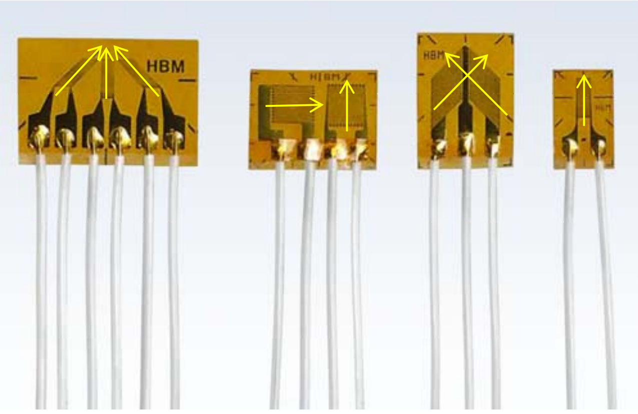
Compensation of common modes with the dummy gauge



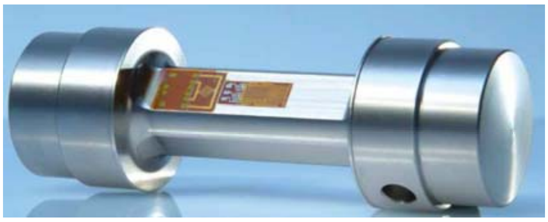
- Temperature is the typical common mode. The compensation can be achieved considering an additional gauge that is not strained by the forces but kept at the same temperature.
- Compensation is achieved in the small variations regime. The method is particularly effective for metallic strain gauges.
- In any case, a reduction of the temperature influence is obtained.

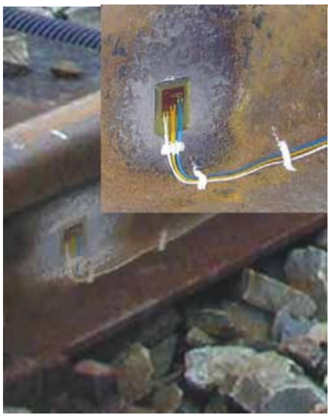
$$R_{attiva} = R_0 \cdot (1 + k \cdot \varepsilon + \alpha \cdot T)$$
$$R_{dummy} = R_0 \cdot (1 + \alpha \cdot T)$$

$$\Delta V = \frac{V}{4} \cdot (\delta_{active} - \delta_{dummy}) =$$
$$= \frac{V}{4} \cdot [(1 + k \cdot \varepsilon + \alpha \cdot T) - (1 + \alpha \cdot T)] = \frac{V}{4} \cdot k \cdot \varepsilon$$



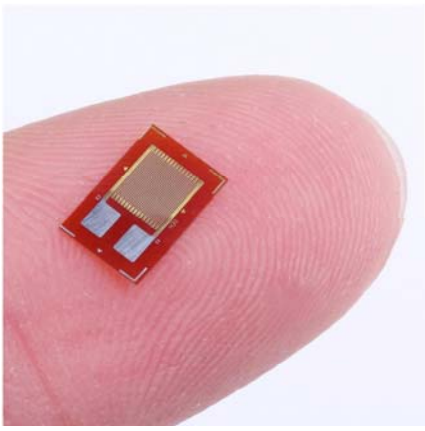
Nominal resistance[1]	Ω	120, 350, 700 or 1,000, depending on SG type
Resistance tolerance[1]	%	± 0.35
with 0.6 mm and 1.5 mm measuring grid length	%	± 1
Gage factor		approx. 2 (stated on the packaging)
Gage factor tolerance[1]	%	± 1
with 0.6 mm and 1.5 mm measuring grid length	%	± 1.5
Temperature coefficient of gauge factor[1]	1/K (1/°F)	$(115 \pm 10) \cdot 10^{-6}$ $((64 \pm 5.5) \cdot 10^{-6})$





Adhesive	Description	Suitable SG	Pot life at room temperature (RT)
Cold curing Z 70 Order No.: 1-Z70	Cyanacrylate adhesive, low viscosity,	optimum: Y, C, LD, LE, V SG residual stress good: K, G	–
for optional use with Z 70 1-BCY01	Accelerator for Z 70		
X 60 Order No.: 1-X60	Methyl metacrylate Two-component adhesive pasty, also suitable for absorbent or uneven surfaces	optimum: Y, C, LD, V SG residual stress good: K, G, LS	ca. 5 minutes

Miniaturized and embedded strain gauges



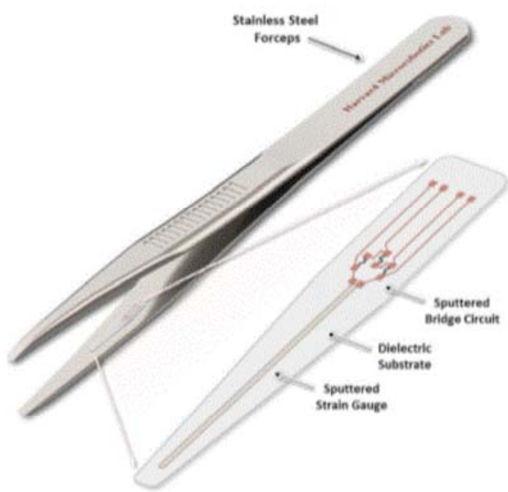


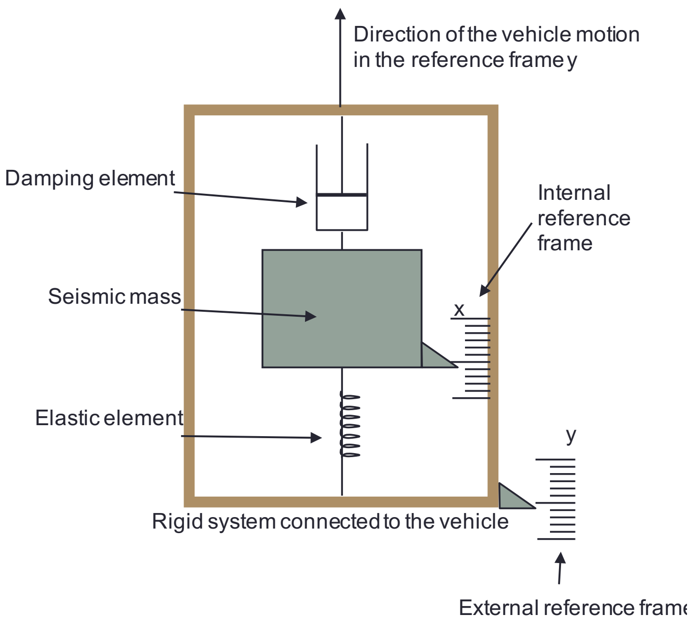
Fig. 1 Conceptual illustration of a strain gauge printed on the surface of stainless steel forceps for pinch force feedback

Sensors of acceleration

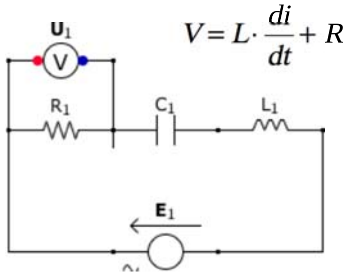
- The measure “on board” of the acceleration respect to an external reference frame is important to control the motion parameters of the vehicle.
- The measure is possible exploiting the second law of Newton F=M*a
  - Then acceleration is evaluated measuring the force acting on a body of mass M
- Acceleration is often measured in units of the average gravitational acceleration (1 g = 9.8 ms^{-2}).
- Accelerometers are used to control the kinetic properties of bodies without the necessity to measure the position with respect to a fixed reference frame.
- Massive applications of accelerometers are found in car industry (airbags control), in electronic games, in smartphones and tablets (inclination sensor, games control,…).
  - The large number of sold pieces accounts for the use of micromachining fabrication techniques and single-chips integration

Accelerometer model

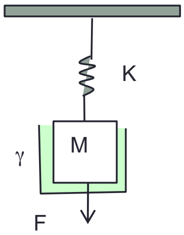
- The accelerometer is a closed mechanical system formed by a mass (called seismic mass), a spring and a damper. The system is tightly connected to the moving vehicle.
- The vehicle moves in a reference frame y, the reference frame of the mechanical system is x.
- The scope of the accelerometer is to measure the acceleration with respect to y through a measure of the kinematic quantities with respect to the reference x.
- The total position of the mass M is (x+y), an inertial force proportional to the second derivative of (x+y) acts on the mass M.
- On the other end, elastic and damping forces acting on the mass depend on the coordinate with respect to the internal reference frame x.



Mechaninal – electrical analogies



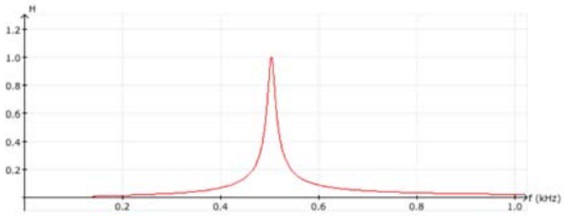
$$V = L \cdot \frac{di}{dt} + R \cdot i + \frac{1}{C} \cdot \int i \cdot dt$$



$$F = M \cdot \frac{d^2x}{dt^2} + \gamma \cdot \frac{dx}{dt} + K \cdot x$$

$$v = \frac{dx}{dt} \Rightarrow F = M \cdot \frac{dv}{dt} + \gamma \cdot v + K \cdot \int v \cdot dt$$

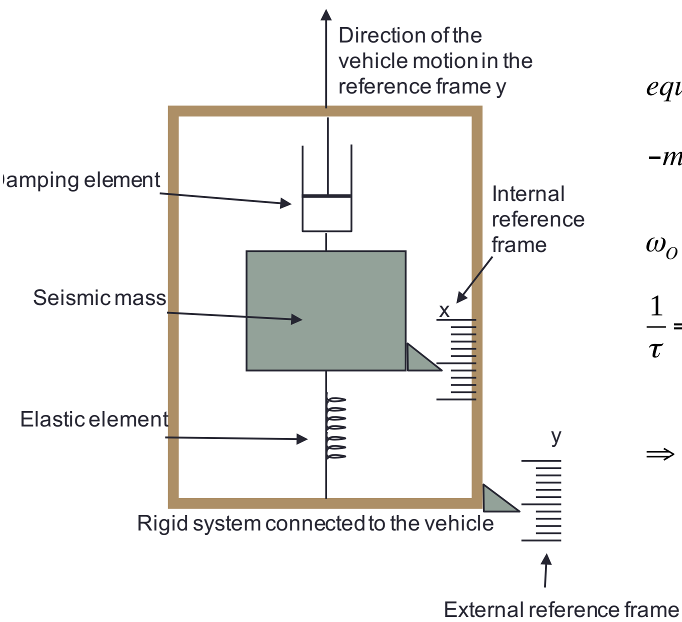
$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}; \quad Q = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$$



$$\omega_0 = \sqrt{\frac{K}{M}}; \quad Q = \frac{1}{\gamma} \cdot \sqrt{M \cdot K}$$

mechanics	*velocity*	*applied force*	*mass*	*Elastic constant*	*friction*
electronics	*current*	*applied voltage*	*inductance*	*Inverse of capacitance*	*resistance*

Equation of motion



equilibrium: inertial force+damping force+spring force=0

$$-m\frac{d^2(x+y)}{dt^2} + \gamma\frac{dx}{dt} + kx = 0 \Rightarrow m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = -m\frac{d^2y}{dt^2}$$

$$\omega_O = \sqrt{\frac{k}{m}} : \quad \text{resonance frequency}$$

$$\frac{1}{\tau} = \frac{\gamma}{m} : \text{damping coefficient}$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{1}{\tau} \cdot \frac{dx}{dt} + \omega_O^2 x = -\frac{d^2y}{dt^2}$$

Frequency response

- The accelerometer is a damped harmonic oscillator, the mass oscillates at the same frequency of the applied force.
- To study the frequency response, let us apply a sinusoidal force.

$$\frac{d^2x}{dt^2}+\frac{1}{\tau}\cdot\frac{dx}{dt}+\omega_O^2 x=-a(t)$$

$$a(t)=a_0\cdot e^{j\omega t}$$

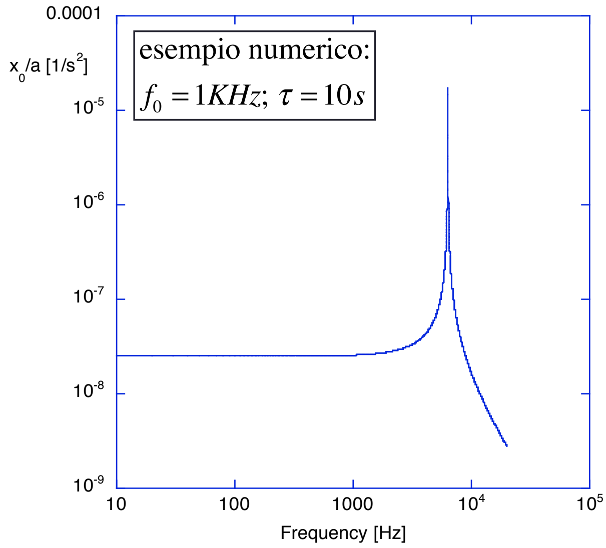
$$\Rightarrow x=x_0\cdot e^{j\omega t};\ \frac{dx}{dt}=j\omega\cdot x_0\cdot e^{j\omega t};\ \frac{d^2x}{dt^2}=-\omega^2\cdot x_0\cdot e^{j\omega t}$$

$$-\omega^2\cdot x_0\cdot e^{j\omega t}+\frac{1}{\tau}\cdot j\omega\cdot x_0\cdot e^{j\omega t}+\omega_O^2 x_0\cdot e^{j\omega t}=-a_0\cdot e^{j\omega t}$$

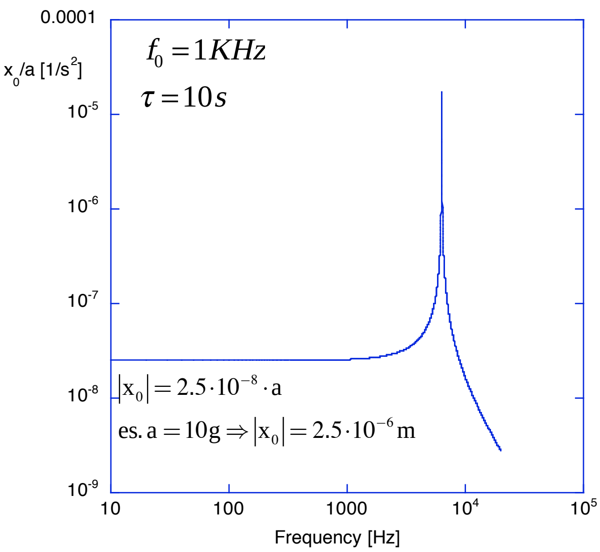
$$x_0\cdot\left[\left(\omega_O^2-\omega^2\right)+j\frac{\omega}{\tau}\right]=-a_0$$

$$|x_0|=\frac{a_0}{\left[\left(\omega_O^2-\omega^2\right)^2+\frac{\omega^2}{\tau^2}\right]^{\frac{1}{2}}}$$

Resonant low-pass behavior



Frequency response



At low frequency, $\omega \ll \omega_0$ the response does not depend on the frequency.

The displacement depends on the elastic force of the spring

$$|x_0|=\frac{a_0}{\omega_O^2}=\frac{M}{K}a_0$$

At the resonance condition, $\omega=\omega_0$, the displacement is

$$x_0=-\frac{a_0}{\omega_0^2}\cdot\tau$$

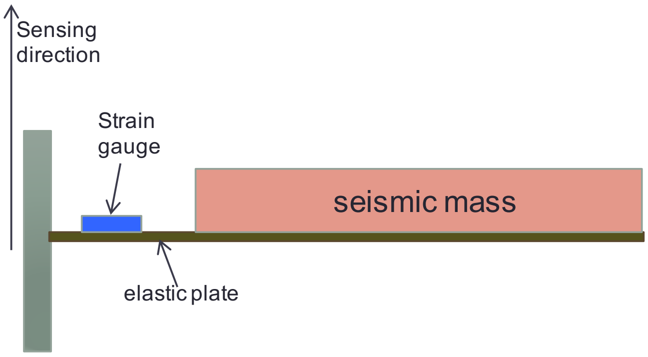
The damping is necessary to ensure the stability of the mechanical system.

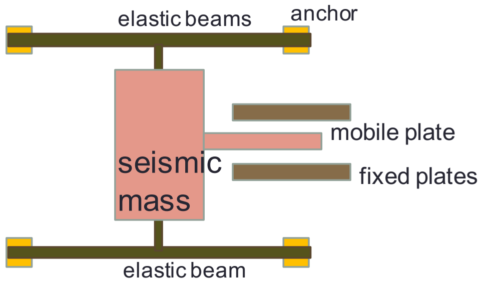
At $\omega \gg \omega_0$ the displacement is inversely proportional to the frequency, and the mechanical system response is dominated by the damping force.

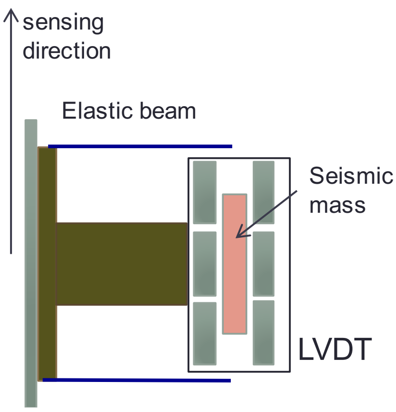
$$x_0=-\frac{a_0}{\left[-\omega^4+\frac{\omega^2}{\tau^2}\right]^{\frac{1}{2}}}$$

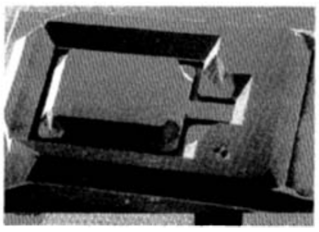
Main transductions

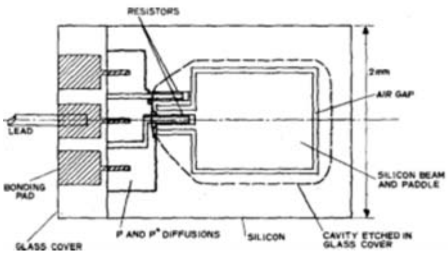
- Strain
  - The seismic mass is mounted at the end of a flexible plate (cantilever)
  - Transduction is accomplished by strain gauges
- displacement
  - The seismic mass is kept suspended by two elastic beams
  - Position is measured by differential capacitors





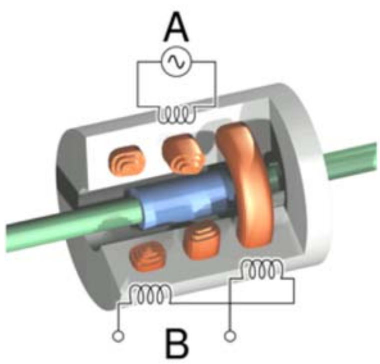


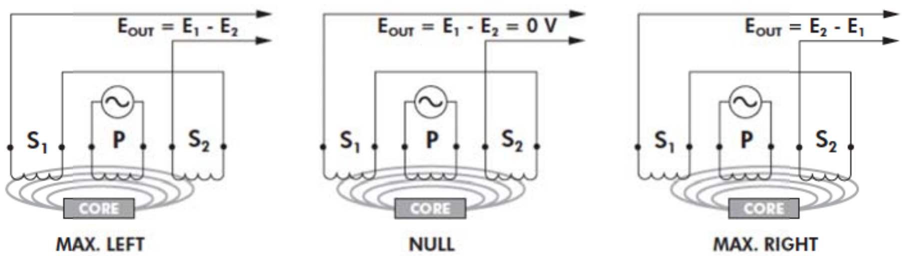




Linear Variable Displacement Transformer (LVDT)

- It is made by one primary coil and two secondary coils wired in counter-phase.
- The ferromagnetic nucleus is free to move and it can be connected to the body whose position has to be sensed.





- V_1-V_2=0 when the nucleus is perfectly symmetrical with respect to the secondary coils. This is the null reference position.
- V_1-V_2 is proportional to the displacement of the nucleus with respect to the null position.
- The frequency of the excitatory signal should be at least 10 times larger than the frequency at which the position changes.
- LVDTs are robust and precise sensors that can be used in mechanical systems.

Example of LVDT

MACRO SENSORS

Input Power	24 V DC (nominal) 15-24 V DC ±10%, 30 mA(nominal)
Full Scale Output	0 to 10 V DC
Output Noise & Ripple	<5 mVrms
Frequency Response (-3dB)	250 Hz (nominal)

PARAMETER	UNIT OF MEASURE	HSER 750 -100	HSER 750 -250	HSER 750 -500	HSER 750 -1000	HSER 750 -2000	HSER 750 -4000	HSER 750 -6000	HSER 750 -10000	HSER 750 -15000	HSER 750 -20000
Nominal Range	Inches	0.100	0.250	0.500	1.00	2.00	4.00	6.00	10.00	15.00	20.00
Nominal Range	Millimeter	2.5	6.3	12.7	25.4	50.8	101.6	152.4	254.0	381.0	508.0
Scale Factor	V/Inch	100	40	20	10	5.0	2.5	1.65	1.0	0.67	0.5
Scale Factor	V/Millimeter	4.0	1.6	0.8	0.4	0.2	0.1	0.06	0.04	0.03	0.02

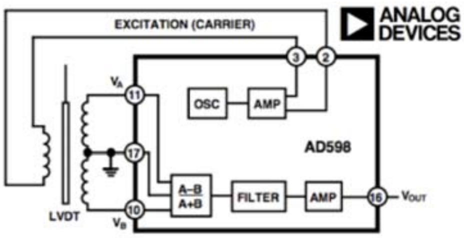
$$\Delta x_{ris} = \frac{v_{noise}}{S} = \frac{5 \cdot 10^{-3}\ V}{4\ \frac{V}{mm}} = 1.25\ \mu m$$

$$n = \frac{\Delta x_{max}}{\Delta x_{ris}} = \frac{2.5\ mm}{1.25\ \mu m} = 2000$$

$$\Delta x_{ris} = \frac{v_{noise}}{S} = \frac{5 \cdot 10^{-3}\ V}{0.02\ \frac{V}{mm}} = 250\ \mu m$$

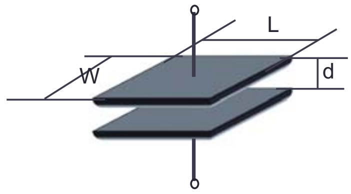
$$n = \frac{\Delta x_{max}}{\Delta x_{ris}} = \frac{508\ mm}{250\ \mu m} = 2032$$

Monolithic LVDT signal conditioner AD598



Capacitive position transducers

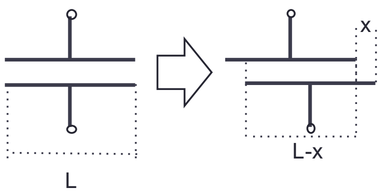
- Capacitance depends on the distance between the plates. if one plate is kept still, the capacitance provides a measure of the position of the other plate.
- Considering a simple plane and parallel capacitor there are two displacement can be measured along two directions.



$$C = \varepsilon \cdot \varepsilon_o \cdot \frac{LW}{d}$$

this formula is valid when the fringe electric field is negligible, namely when the plates are perfectly parallel and d≪L, W

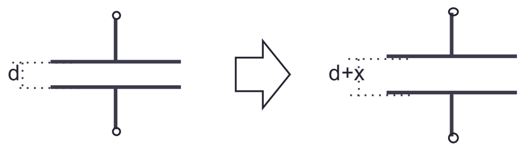
transversal shift



$$C \approx \epsilon_r \epsilon_0 \frac{W}{d}(L - x)$$

this condition violates the plane and parallel capacitor condition: approximated formula

parallel shift



$$C \approx \epsilon_r \epsilon_0 \frac{WL}{d + \delta}$$

Taylor expansion:

$$C(d+\delta) = C(d) + \delta \frac{dC}{d\delta} + \frac{\delta^2}{2}\frac{d^2C}{d\delta^2} + \dots$$

$$C(d+\delta) \approx= \epsilon_r \epsilon_0 \frac{A}{d}\left(1 - \frac{\delta}{d} + \frac{\delta^2}{d^2}\right)$$

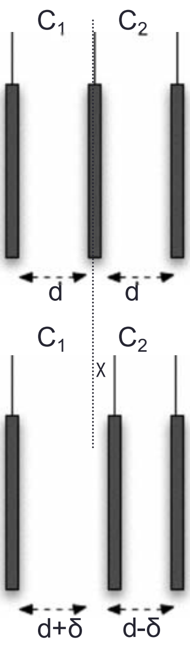
Differential capacitor

- The differential measurement of displacement can be accomplished by a couple of capacitors with a common plate.
- In this arrangement, a change of position of the common plate increases of one of the capacitances and decreases the other.
- Taylor expansion of the difference of the capacitances gives:

$$\Delta C = C_2 - C_1 \approx \varepsilon\varepsilon_o \frac{A}{d-\delta} - \varepsilon\varepsilon_o \frac{A}{d+\delta}$$
$$= \varepsilon\varepsilon_o \frac{A}{d}\left(1+\frac{\delta}{d}+\frac{\delta^2}{d^2}\right) - \varepsilon\varepsilon_o \frac{A}{d}\left(1-\frac{\delta}{d}+\frac{\delta^2}{d^2}\right)$$
$$= \varepsilon\varepsilon_o \frac{A}{d}\cdot\frac{2\cdot\delta}{d} = C_0\cdot\frac{2\cdot\delta}{d}$$

- The non linearity appears at the third-order, the differential capacitor increases the linearity and doubles the sensitivity.
- The position measurement is not perturbed by the applied voltage.
  - Given a plane capacitor, biased with a voltage V_S the plates attract each other with a force:

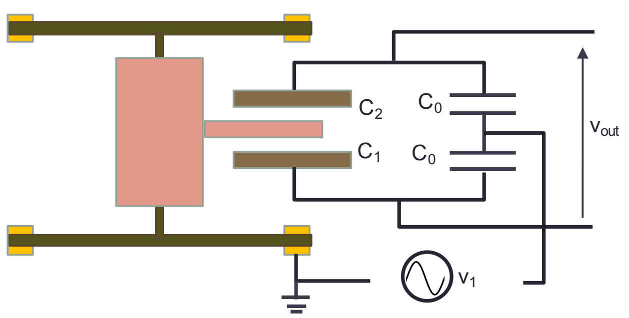
$$F = -\epsilon_0 \frac{S}{2d^2} V_s^2$$

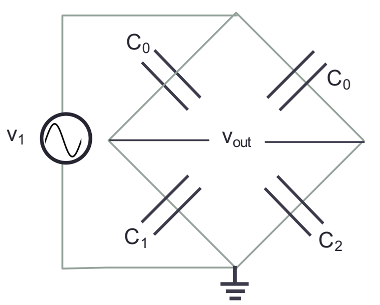


Differential capacitor in a Wheatstone bridge

- The acceleration can be measured in a Wheatstone bridge configuration.
- The seismic mass is connected to the common plate of a differential capacitor

$$C_1 = \epsilon\frac{S}{x_0+\Delta x} = \epsilon\frac{S}{1+\frac{\Delta x}{x_0}} = \epsilon\frac{S}{x_0(1+\delta)} = \frac{C_0}{1+\delta}$$
$$C_2 = \epsilon\frac{S}{x_0-\Delta x} = \epsilon\frac{S}{1-\frac{\Delta x}{x_0}} = \epsilon\frac{S}{x_0(1-\delta)} = \frac{C_0}{1-\delta}$$
$$C_0 = \epsilon\frac{S}{x_0}$$
$$\delta = \frac{\Delta x}{x_0} = \frac{M}{Kx_0}a$$





$$v_{out} = v_i\cdot\left(\frac{1/j\omega C_1}{1/j\omega C_1 + 1/j\omega C_0} - \frac{1/j\omega C_2}{1/j\omega C_2 + 1/j\omega C_0}\right) =$$
$$= v_i\cdot\left(\frac{1/C_1}{1/C_1+1/C_0} - \frac{1/C_2}{1/C_2+1/C_0}\right) =$$
$$= v_i\cdot\left(\frac{(1+\delta)/C_0}{(1+\delta)/C_0+1/C_0} - \frac{(1-\delta)/C_0}{(1-\delta)/C_0+1/C_0}\right) =$$
$$= v_i\cdot\left(\frac{1+\delta}{2+\delta} - \frac{1-\delta}{2-\delta}\right) =$$
$$= v_i\cdot\frac{2\delta}{4-\delta^2}$$

$$\delta \ll 1 \to v_{out} = v_i\frac{\delta}{2}$$
$$v_{out} = v_i\frac{M}{2Kx_0}a$$

Signal frequency

Let us consider the current in the capacitors

$$i_c(t) = \frac{dQ}{dt} = \frac{d(Cv_c)}{dt} = C(x(t))\frac{dv_c}{dt} + v_c\frac{dC(x(t))}{dx}\frac{dx}{dt}$$

$$v_c = V_0 exp(j\omega t) \rightarrow i_c(t) = j\omega v_c C(x(t)) + v_c\frac{dC(x(t))}{dx}\frac{dx}{dt}$$

$$i_c = v_c\left(j\omega v_c C(x(t)) + \frac{dC(x(t))}{dx}\frac{dx}{dt}\right)$$

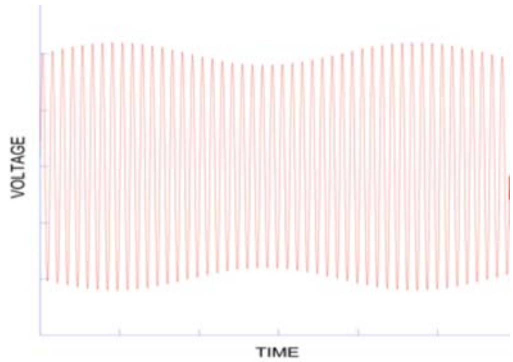
The electric frequency has to be large enough to make the velocity contribution negligible.

In case of d.c.

$$v_c = V_0 \quad and \quad \frac{v_c}{dt} = 0$$

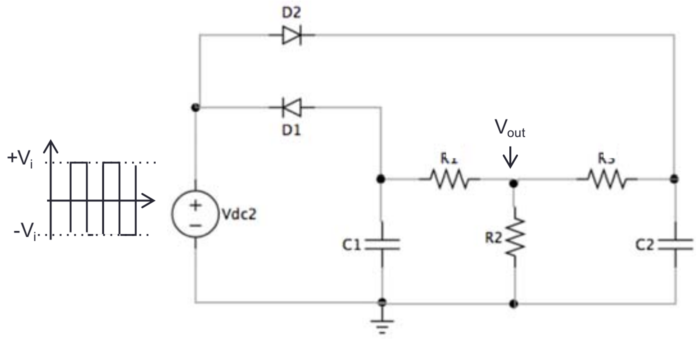
$$i_c = V_0\frac{dC}{dx}\frac{dx}{dt}$$

instead of the position, the velocity of the mass is measured.

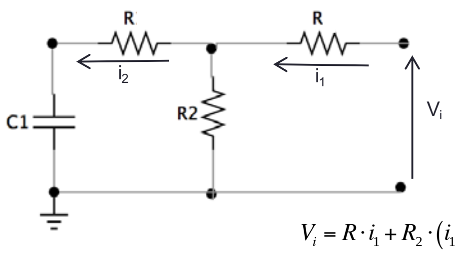


The acceleration signal is a amplitude modulation of the electric signal at frequency ω.
A demodulation is necessary to eliminate the high frequency and to get x(t) and a(t)

Lion Bridge



Equivalent circuit at V_i>0



$$V_i = R\cdot i_1 + R_2\cdot(i_1 - i_2)$$

$$v_{C1} = R\cdot i_2 + R_2\cdot(i_2 - i_1)$$

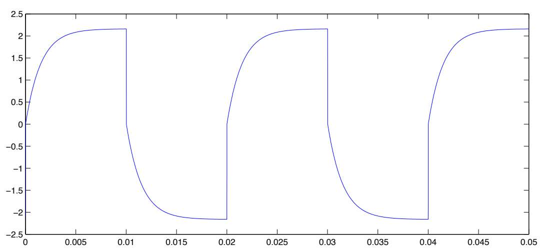
$$i_2 = -C_1\frac{dv_{C1}}{dt}$$

$$V_{C1}(t=0) = -V_i$$

Solving the equation:

$$v_{OUT}^{+} = V_i\cdot\frac{R_2}{R+R_2}\cdot\left[1-\exp\left(-\frac{R+R_2}{R\cdot(R+2\cdot R_2)}\frac{t}{C_1}\right)\right]$$

$$v_{OUT}^{-} = -V_i\cdot\frac{R_2}{R+R_2}\cdot\left[1-\exp\left(-\frac{R+R_2}{R\cdot(R+2\cdot R_2)}\frac{t}{C_2}\right)\right]$$



Mean voltage:

$$V_{med} = \frac{1}{T}\left[\int_0^{T/2} v_{OUT}^{+}\,dt + \int_{T/2}^{T} v_{OUT}^{-}\,dt\right]$$

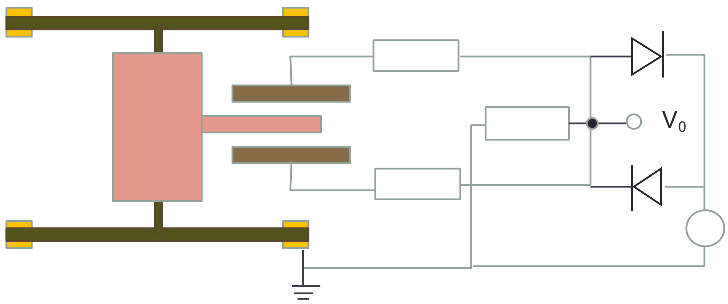
if R»R₂ the argument of the exponential becomes:

$$V_{med} = V_i\cdot\frac{R_2R\cdot(R+2\cdot R_2)}{(R+R_2)\cdot(R+R_2)}\cdot\frac{1}{T}\left\{C_1\cdot\left[1-\exp\left(-\frac{T}{RC_1}\right)\right]-C_2\cdot\left[1-\exp\left(-\frac{T}{RC_2}\right)\right]\right\}$$

$$if\ T»\frac{1}{RC_1};\frac{1}{RC_2} \Rightarrow V_{med} = V_i\cdot\frac{R_2R\cdot(R+2\cdot R_2)}{(R+R_2)\cdot(R+R_2)}\cdot\frac{1}{T}\cdot(C_1-C_2) \Rightarrow V_{med} = K\cdot(C_1-C_2)$$

K is few mV/pF

Cantilever accelerometer differential capacitor in a Lion bridge



Variation of distance between plates

Let us consider a parallel shift of the seismic mass.

$$x_0 = 2\ \mu m$$

$$\Delta z = 25 \cdot 10^{-9} m = 25 \cdot 10^{-3} \mu m$$

$$\delta = \frac{\Delta z}{x_0} = \frac{25 \cdot 10^{-3}}{2} = 25 \cdot 10^{-3}$$

In a Lion bridge with a sensitivity K=10 mV/pF

$$V_{O,rms} = k \cdot (C_1 - C_2) = k \cdot \left(\varepsilon \frac{A}{x_0 - \Delta z} - \varepsilon \frac{A}{x_0 + \Delta z} \right) \approx k \cdot \varepsilon \frac{A}{x_0} \cdot \frac{2 \cdot \Delta z}{x_0} = k \cdot C_0 \cdot 2 \cdot \frac{\Delta z}{x_0} = C_0 \cdot 2 \cdot \delta = 0.8\ mV$$

MEMS: MicroElectroMechanical Systems

- Integrated sensors
  - Silicon technology (microelectronics) enables the fabrication of integrated systems where both the sensitive element and the electronics are integrated in the same chip.
  - Some milestones:
    - 1967 anistropic silicon etching
    - 1970 bulk etched silicon as pressure sensor
    - 1979 micromachined ink-jet nozzle (HP)
    - 1983 integrated pressure sensor (Honeywell)
    - 1985 airbag crash sensor (sensonor)
    - 1988 Rotary electrostatic motors (Berkeley)
    - 1993 Digital mirror displays (Texas Instr.)
    - 1993 High volume production of accelerometer (Analog Device)
    - 1994 Deep reactive ion etching (Bosch)


420 PROCEEDINGS OF THE IEEE, VOL. 70, NO. 5, MAY 1982

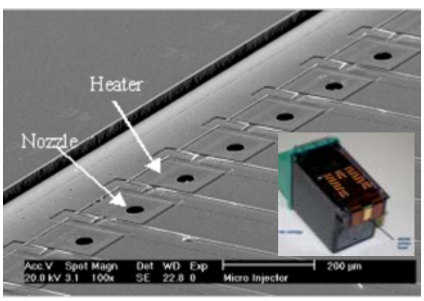
Silicon as a Mechanical Material

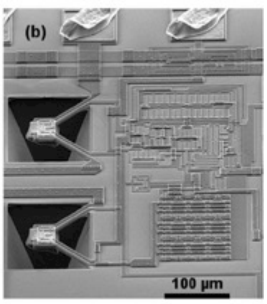
KURT E. PETERSEN, MEMBER, IEEE

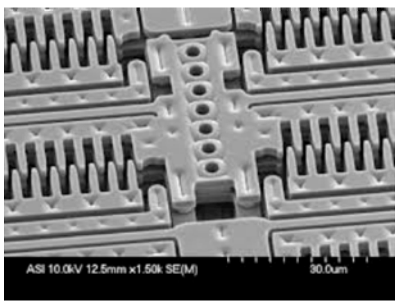
Abstract—Single-crystal silicon is being increasingly employed in a variety of new commercial products not because of its well-established electronic properties, but rather because of its excellent mechanical properties. In addition, recent trends in the engineering literature indicate a growing interest in the use of silicon as a mechanical material with the ultimate goal of developing a broad range of inexpensive, batch-fabricated, high-performance sensors and transducers which are easily interfaced with the rapidly proliferating microprocessor. This review describes the advantages of employing silicon as a mechanical material, the relevant mechanical characteristics of silicon, and the processing techniques which are specific to micromechanical structures. Finally, the potentials of this new technology are illustrated by numerous detailed examples from the literature. It is clear that silicon will continue to be aggressively exploited in a wide variety of mechanical applications complementary to its traditional role as an electronic material. Furthermore, these multidisciplinary uses of silicon will significantly alter the way we think about all types of miniature mechanical devices and components.

miniaturized mechanical devices and components must be integrated or interfaced with electronics such as the examples given above.

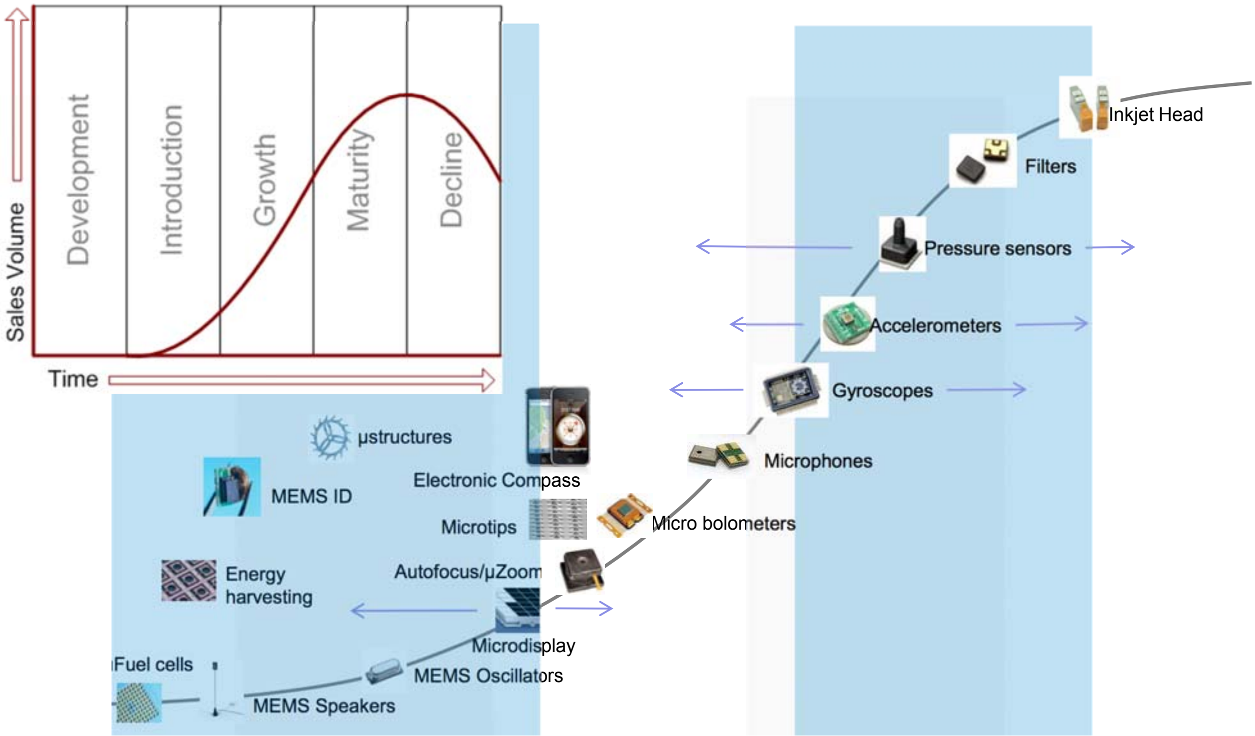
The continuing development of silicon micromechanical applications is only one aspect of the current technical drive toward miniaturization which is being pursued over a wide front in many diverse engineering disciplines. Certainly silicon microelectronics continues to be the most obvious success in the ongoing pursuit of miniaturization. Four factors have played crucial roles in this phenomenal success story: 1) the active material, silicon, is abundant, inexpensive, and can now be produced and processed controllably to unparalleled standards of purity and perfection; 2) silicon processing itself is based on very thin deposited films which are highly amenable to miniaturization; 3) definition and reproduction of the



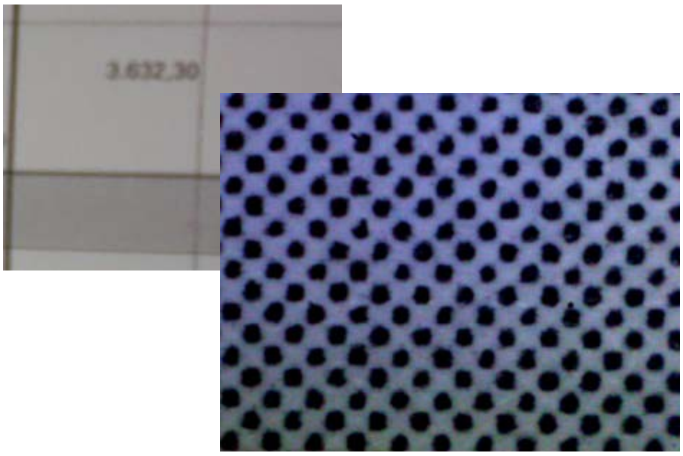




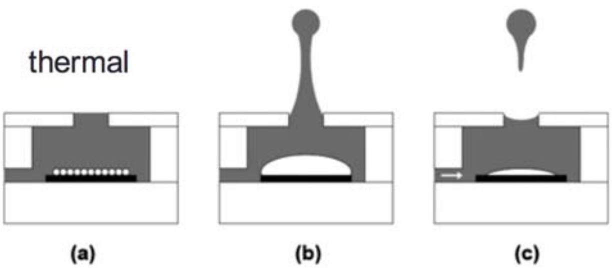
MEMS
Development state of the art



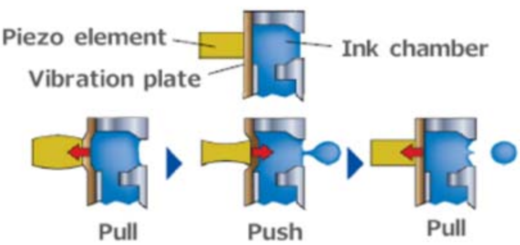
example 1: ink-jet printer

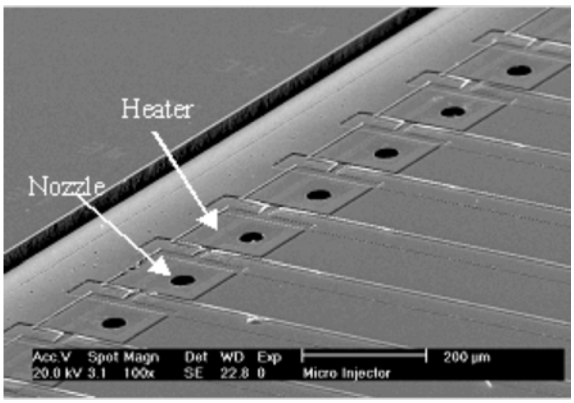


dot size: 9600 dpi = 2 μm

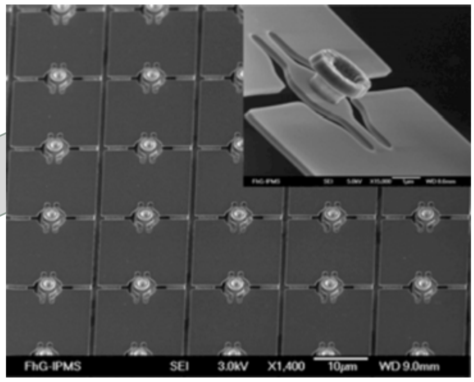


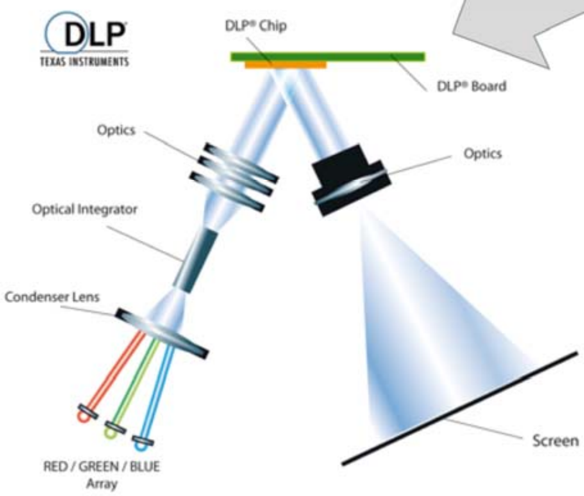
pressure



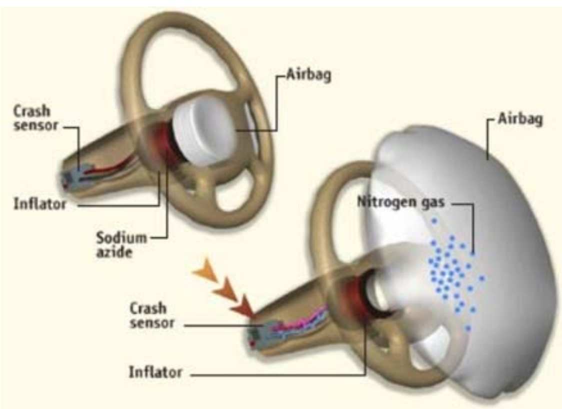


example 2: micromirror array





Car airbags



The signal of the accelerometer is used to trigger a spark that ignites the reaction of a quantity of sodium azide.
This is a solid at room temperature.
Every two molecules of sodium azide, three molecules of nitrogen are produced. Then the air pressure inside the bag quickly increases.

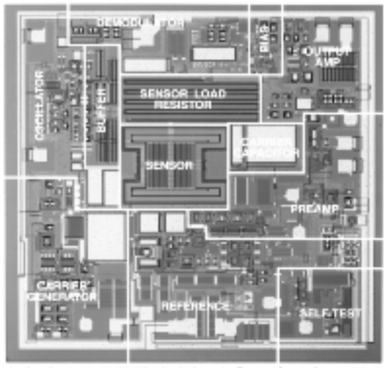
$$2NaN_3 \rightarrow 2Na + 3N_2$$

Metallic sodium is removed by an additional reaction with potassium nitrate. This reaction produces additional nitrogen. Further wastes are removed by a reaction with silicon oxide that results in stable and non toxic alkaline silicates.

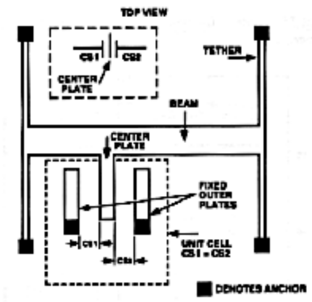
MEMS accelerometer

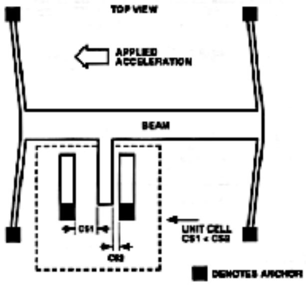
- Several micromachined accelerometers are available in the market, as an example let us consider the ADXL50 aimed at controlling airbags.

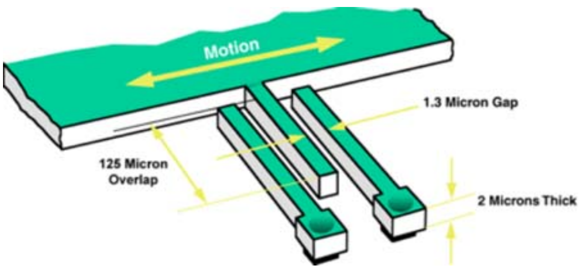
capacitive transducers
interdigitated capactitors to increase the total capacitance

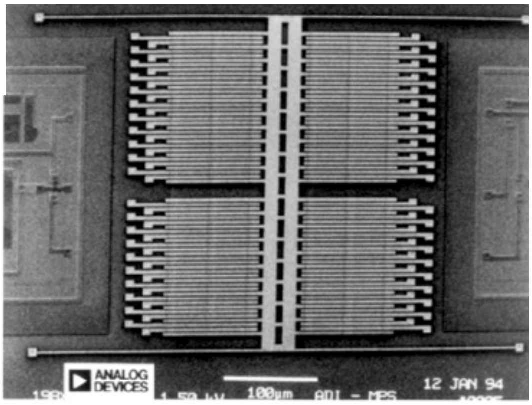


Analog Devices' ADXL-50, the industry's first surface micromachined accelerometer, includes signal conditioning on chip.

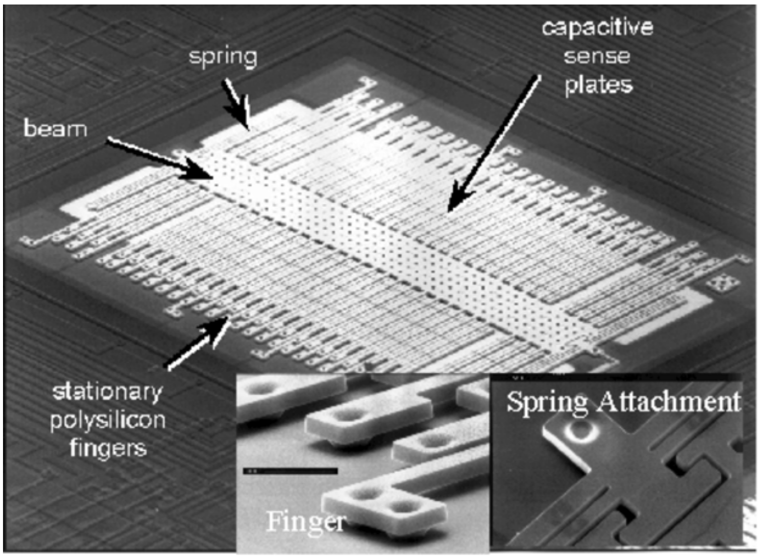


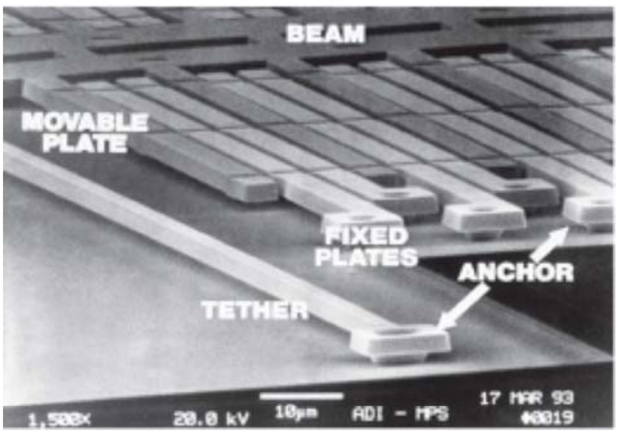


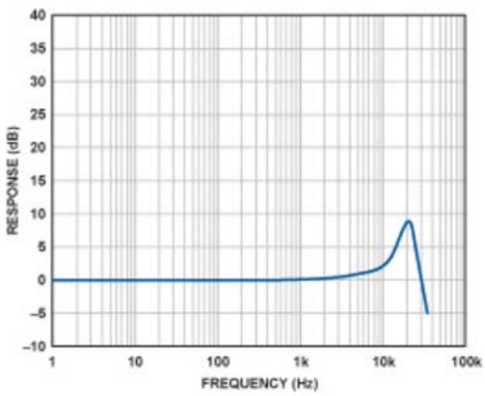




ADXL50 close view

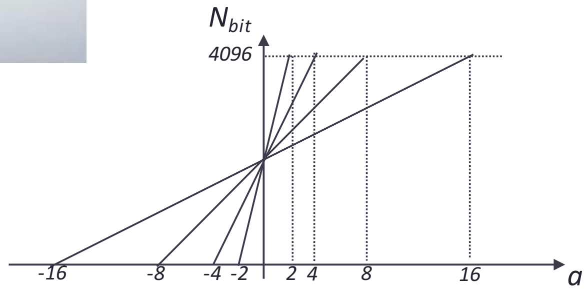






Bosch BMA423

Parameter	Technical data
Digital resolution	12 bit
Resolution (in ±2g range)	0.98 mg
Measurement ranges (programmable)	±2 g, ±4 g, ±8 g, ±16 g
Sensitivity (calibrated)	±2 g: 1024 LSB/g ±4 g: 512 LSB/g ±8 g: 256 LSB/g ±16 g: 128 LSB/g



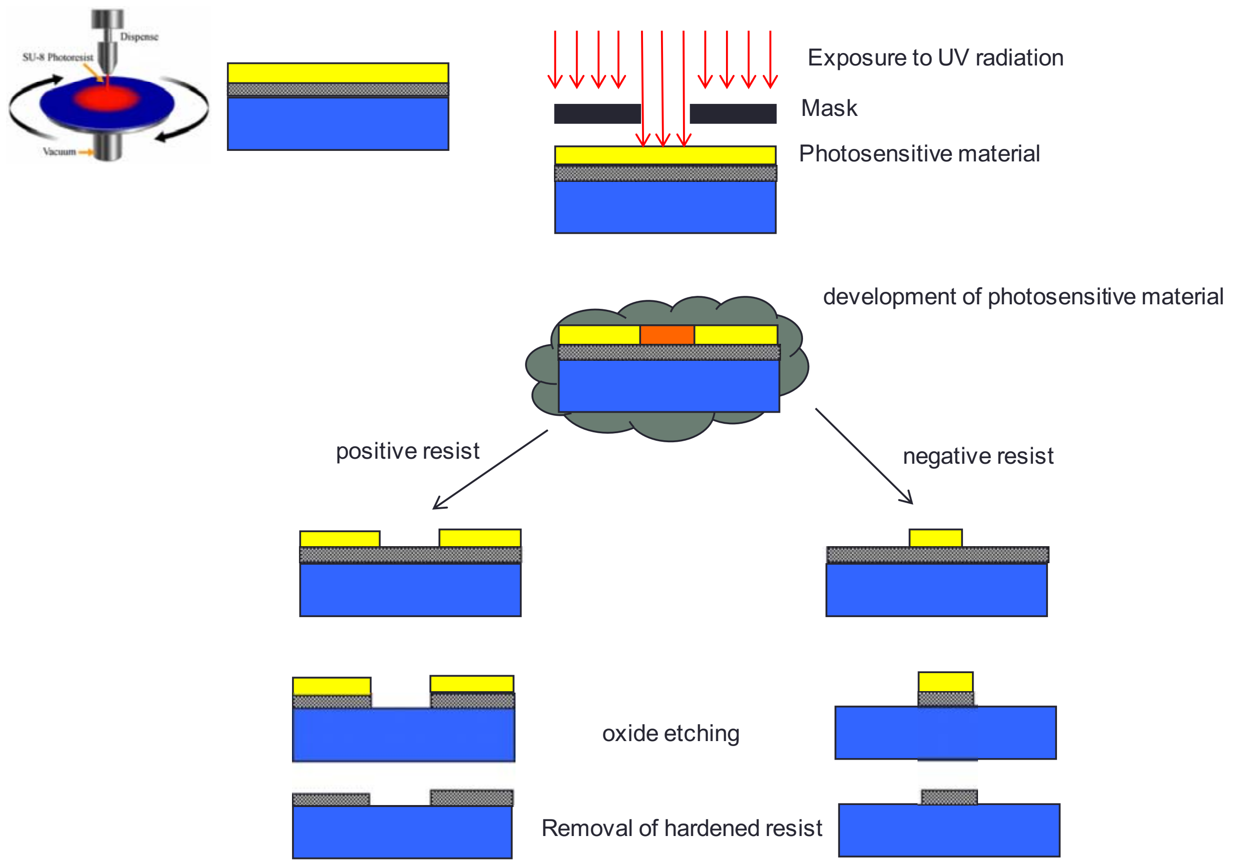
$$S_{\pm 2g} = \frac{4096}{4} = 1024\ \frac{bit}{g} \qquad ris_{\pm 2g} = \frac{1}{1024} = 0.98\ mg$$

$$S_{\pm 4g} = \frac{4096}{8} = 512\ \frac{bit}{g} \qquad ris_{\pm 4g} = \frac{1}{512} = 1.95\ mg$$

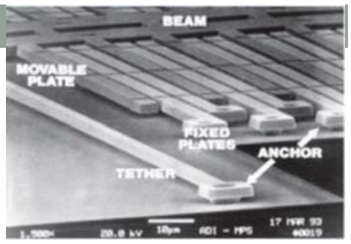
$$S_{\pm 8g} = \frac{4096}{16} = 256\ \frac{bit}{g} \qquad ris_{\pm 8g} = \frac{1}{256} = 3.90\ mg$$

$$S_{\pm 16g} = \frac{4096}{32} = 128\ \frac{bit}{g} \qquad ris_{\pm 16g} = \frac{1}{128} = 7.81\ mg$$

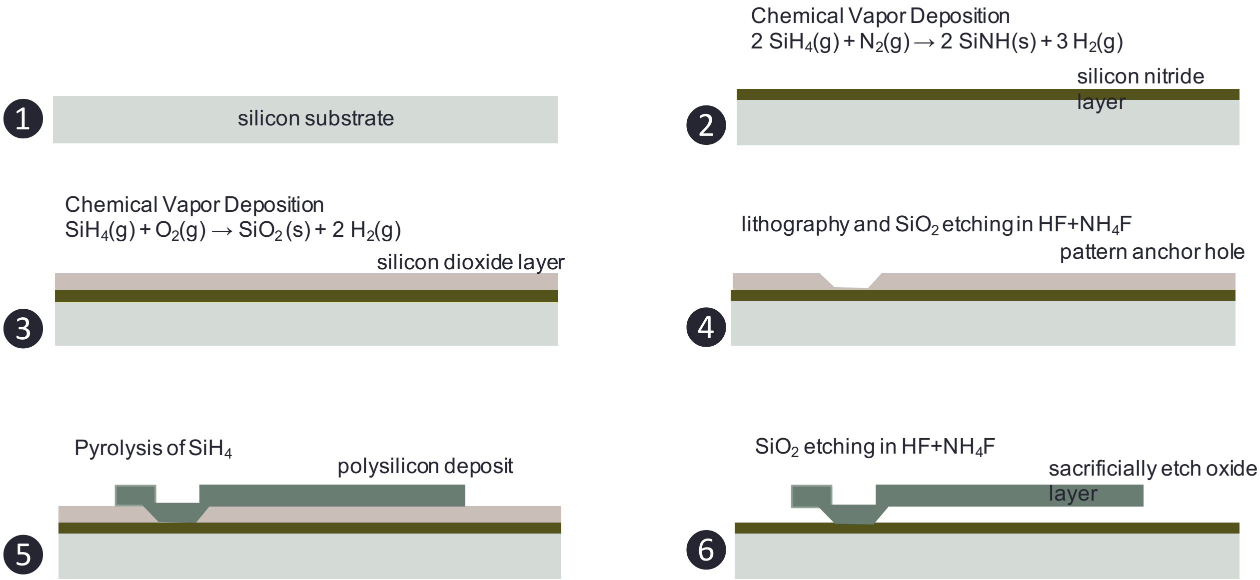
Principle of photo-lithography



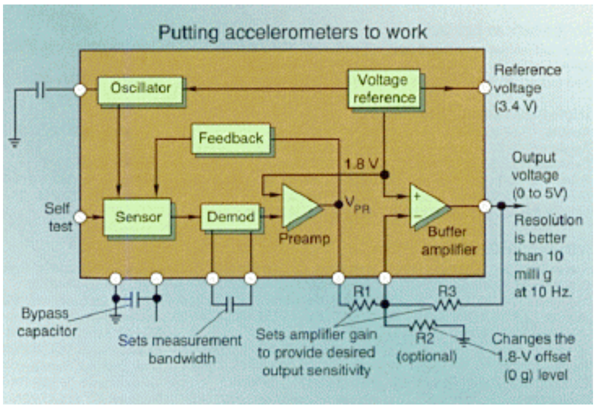
Steps for the fabrication of suspended beams surface micromachining



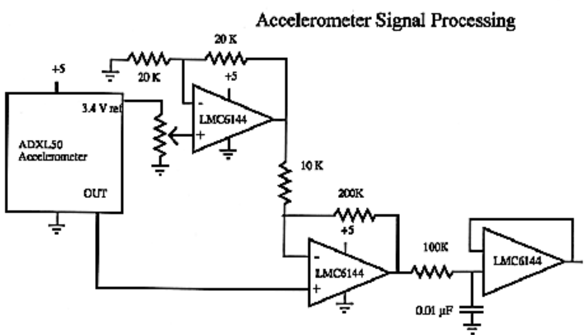
- the central core of micromachining is the selective removal of materials exploiting the affinities of chemical reactions

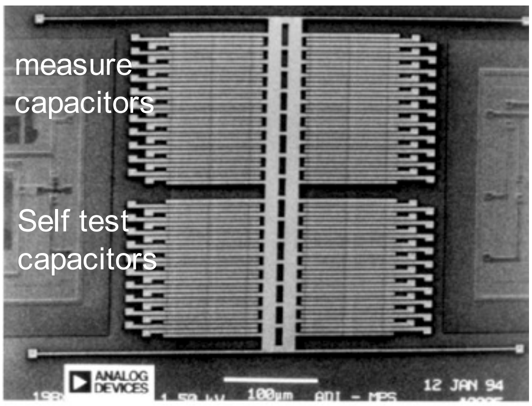


ADXL50 integrated electronics



Comparator to trigger the airbag explosion.



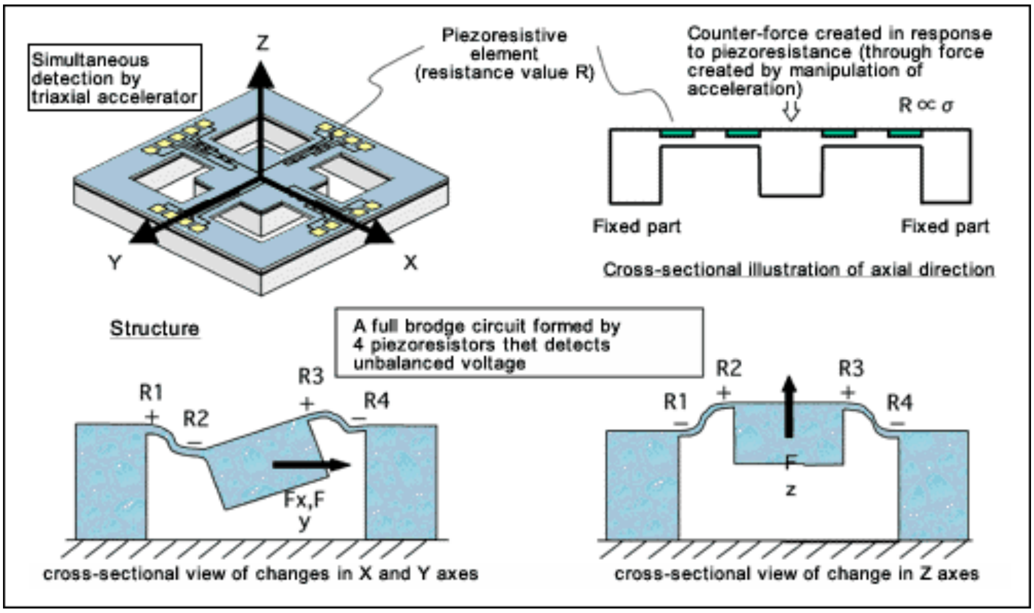


Self-test
a group of single capacitors, not differential, is used to test the accelerometer.
Given a plane capacitor, biased with a voltage V_S the plates attract each other with a force:

$$F = -\epsilon_0 \frac{S}{2d^2} V_s^2$$

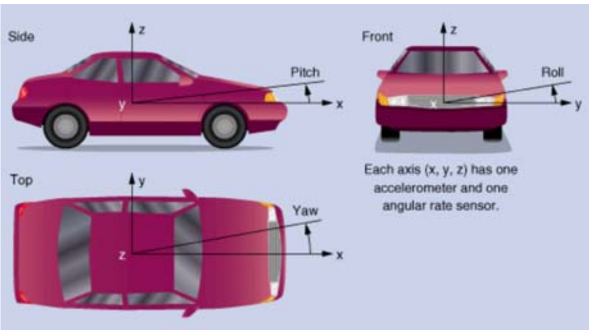
This induces an acceleration a=F/M to the seismic mass. The measure of the output signal allows for calibration of the sensor.

3D accelerometer



Angular speed sensor: gyroscope

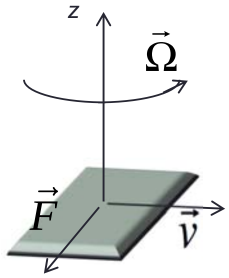
- The measure of the angular speed is important for:
  - inertial drive of vehicles (complemented by accelerometers)
    - control of yaw, pitch and roll
  - Consumer electronics
    - videogames



Angular speed sensors are improperly called gyroscopes
MEMS gyroscopes can be based on mechanical or optical effects, here a mechanical gyroscope based on Coriolis force is illustrated.

Coriolis force

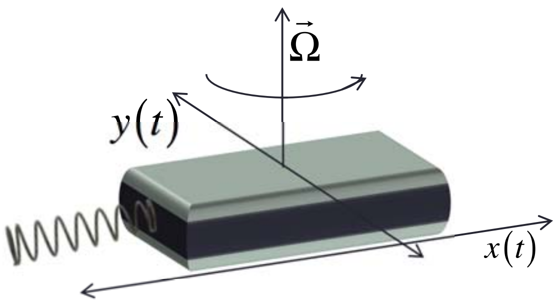
- The Coriolis force is an apparent force acting on a mass that is moving with a velocity v in a reference frame rotating with angular velocity Ω



$$\vec{F} = 2 \cdot m \cdot \vec{v} \times \vec{\Omega}$$

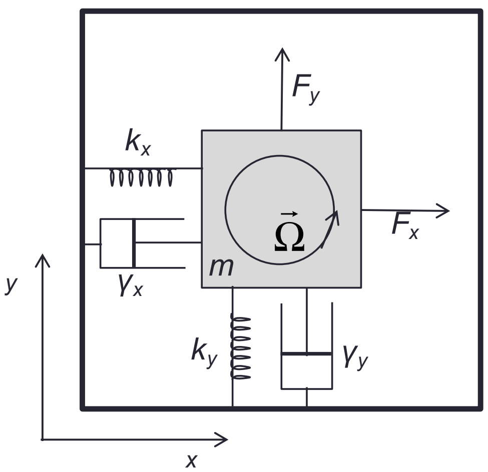
In the inertial frame

- Coriolis force based gyroscope



A mass is kept in oscillation along the direction x, a rotation around the axis z gives rise to a Coriolis force oriented along the direction y. The force is also oscillating and the measure of the magnitude of the oscillation along y gives a measure of the angular velocity.

Gyroscope model



- The motion along x is actuated by the force F_x. This may be a sinusoidal electrostatic force. To maximize the displacement, the frequency of F_x is tuned to the resonance frequency of system.

$$x = x_0 \cdot cos(\omega_x t)$$

- The Coriolis force oriented along y is (Ω constant):

$$F = 2 \cdot m \cdot \Omega \cdot \frac{dx}{dt} = -2 \cdot m \cdot \Omega \cdot x_0 \cdot \omega_x \cdot sin(\omega_x t)$$

motion equation:

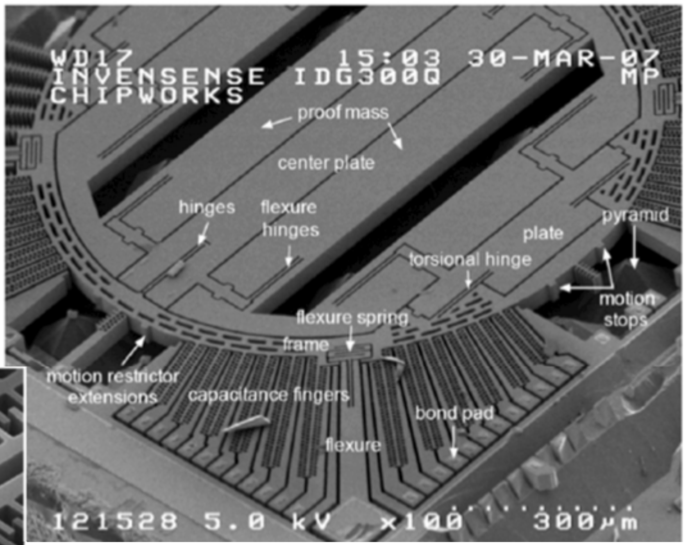
$$m\frac{d^2y}{dt^2} + \gamma \cdot \frac{dy}{dt} + k \cdot y = -2 \cdot m \cdot \Omega \cdot x_0 \cdot \omega_x \cdot sin(\omega_x \cdot t) \Rightarrow \frac{d^2y}{dt^2} + \frac{1}{\tau} \cdot \frac{dy}{dt} + \omega_O^2 y = -2 \cdot \Omega \cdot x_0 \cdot \omega_x \cdot sin(\omega_x \cdot t)$$

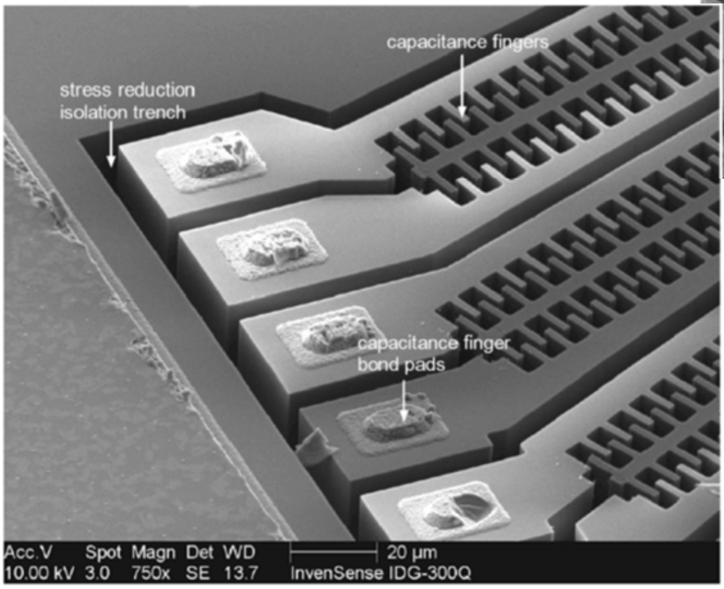
da cui: $y = y_0 \cdot sin(\omega_x \cdot t + \varphi)$

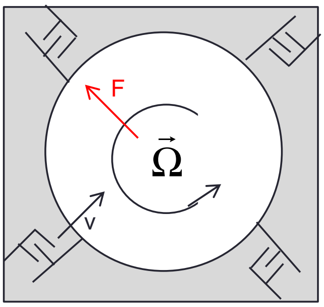
con: $y_0 = \frac{-2 \cdot \Omega \cdot x_0 \cdot \omega_x}{\left[(\omega_0^2 - \omega_x^2)^2 + \left(\frac{\omega_x}{\tau}\right)^2\right]^{1/2}}$

ω_x is the resonance frequency along x
ω_0 is the resonance frequency along y

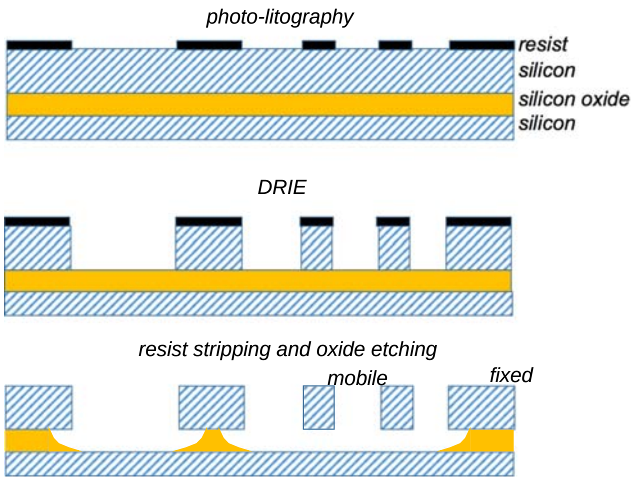
example of MEMS gyroscope



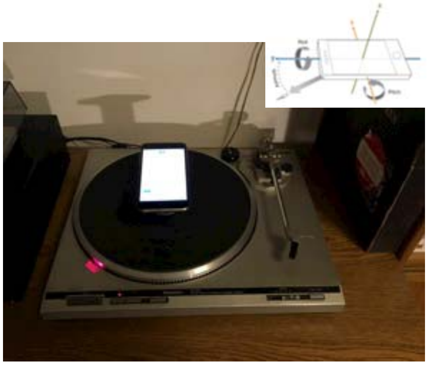


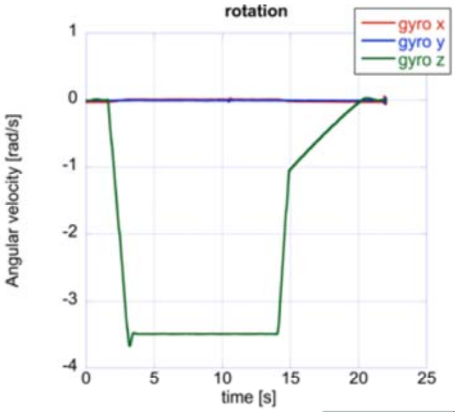


- La fabbricazione del giroscopio richiede ulteriori processi tecnologici
- Reactive Ion Etching
  - Rimozione selettiva del solido attraverso la reazione con specie gassose ionizzate ad alta temperatura
  - Il RIE consente di scavare prevalentemente nella direzione del campo elettrico.
    - Esafluoruro di zolfo (SF_6) per il silicio
  - Deep RIE (Bosch)
    - Deposizione di un materiale inerte (ottafluorociclobutano simile al Teflon) sulle pareti per diminuire la erosione ed aumentare la profondità della struttura
- Silicon-On-Insulator wafer
  - Il SOI consiste in una superficie di silicio cristallino sopra uno strato di ossido.



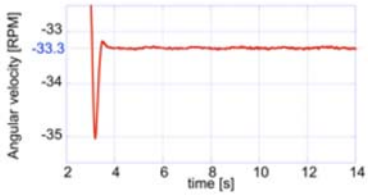
Turntable kinematics

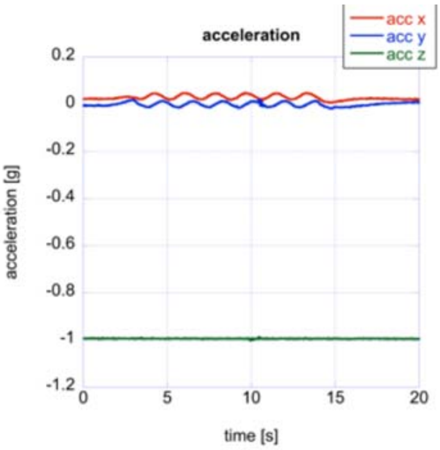




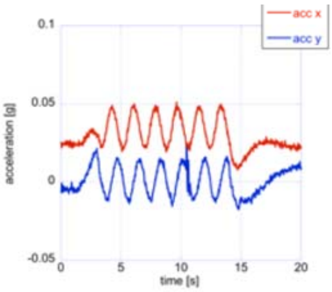
$$v_{ang}\,[RPM] = \frac{60}{2\pi}\cdot\omega\left[\frac{rad}{s}\right]$$

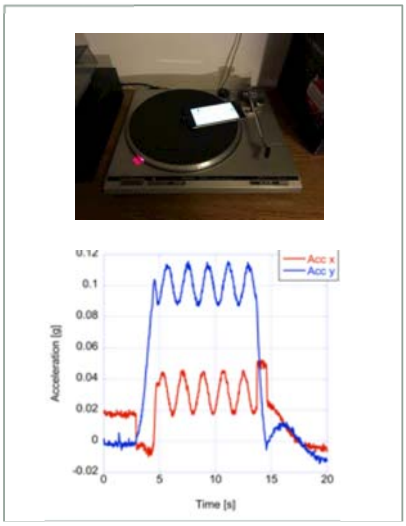
$$33.3\ RPM = 3.37\ \frac{rad}{s}$$





$$\vec{a} = \omega^2 \cdot \vec{r}$$





Pressure

- The pressure is the force exerted by a fluid (liquid or gas) per unit of surface of the container. Pressure is measured in Pascal: 1 Pa= 1 N/m^2.
- In case of gas, the pressure is proportional to the temperature and the density of molecules (perfect law of gases)

$$P_{gas} = \frac{n}{V}RT$$

- 1 Pa is small with respect to commonly experienced pressures. For gases a number of alternative units are in use.

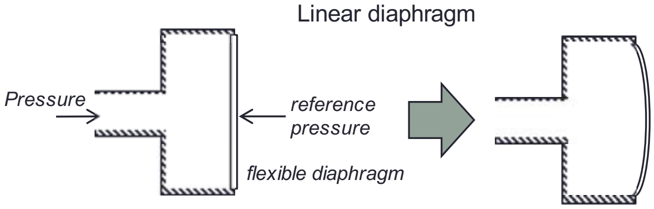
atmosphere (atm)	101325 Pa
bar	10^5 Pa
dyne/cm^2	0.1 Pa
Kg/m^2	9.80 Pa
Kg/cm^2	$9.806 \cdot 10^4$ Pa
millibar	100 Pa
mm Hg (torr)	133.32 Pa
psi (pound/inch^2)	$6.895 \cdot 10^3$ Pa

$$1\ atm = 760\ mm_{Hg} = 101325\ Pa = 1013\ mbar$$

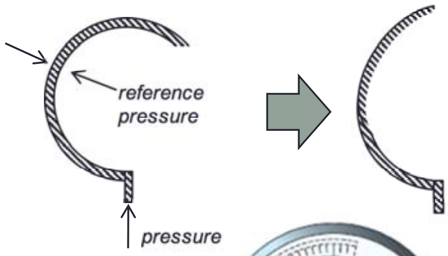
Principles of pressure sensors

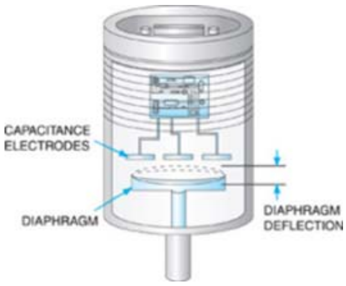
- Pressure is detected with mechanical systems containing a flexible diaphragm.
- Two pressures are applied to the faces of the membrane. One is a reference pressure and the other is the pressure to be measured.
- The equilibrium of forces induces a deformation of the membrane shape. The deformation can be visually detected or measured by suitable strain gauges or displacement sensors.

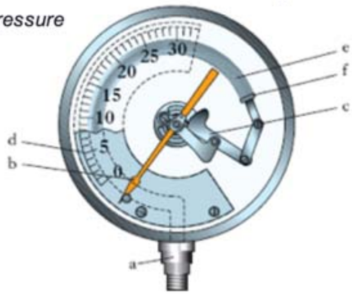
Linear diaphragm



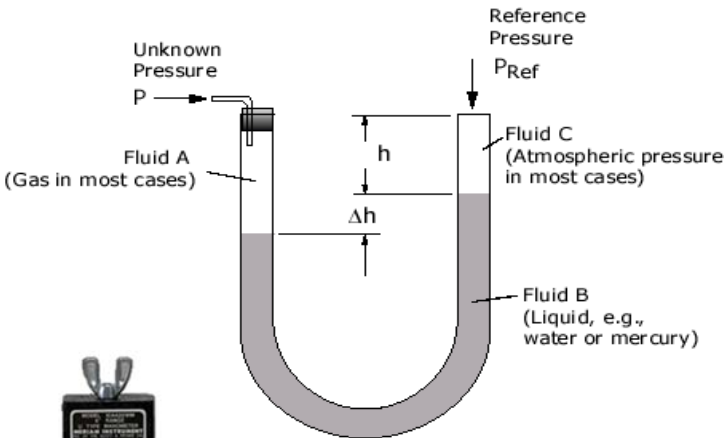
Circular diaphragm: Bourdon tube

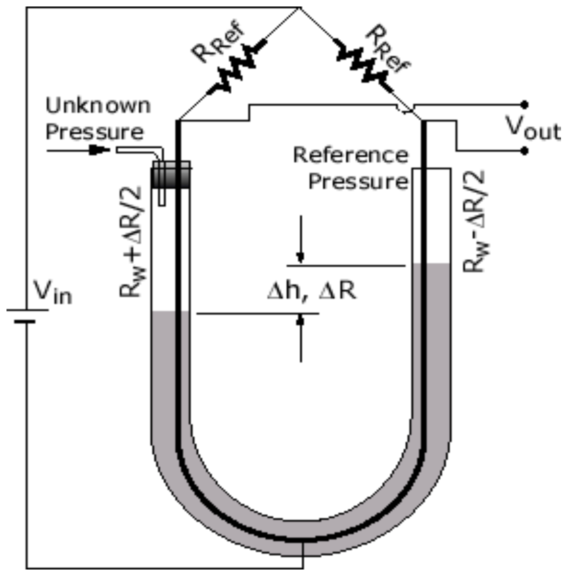




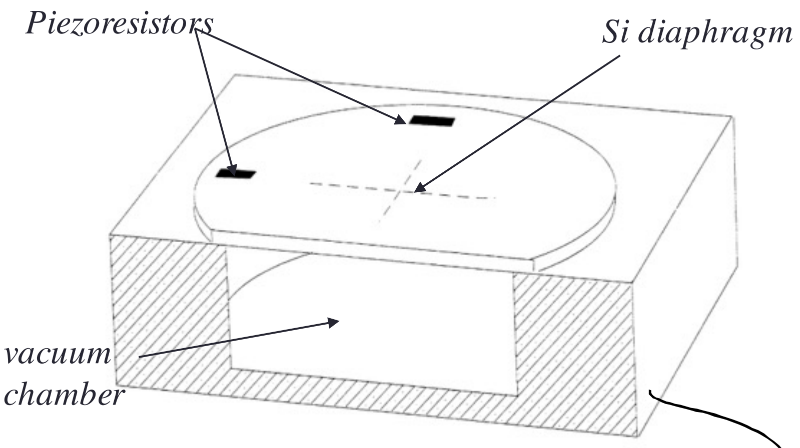


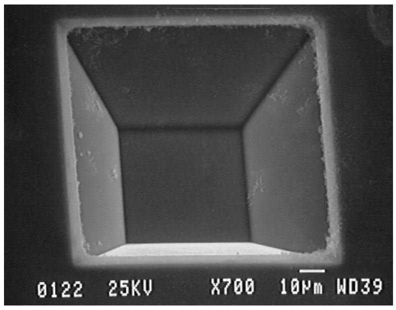
U-tube

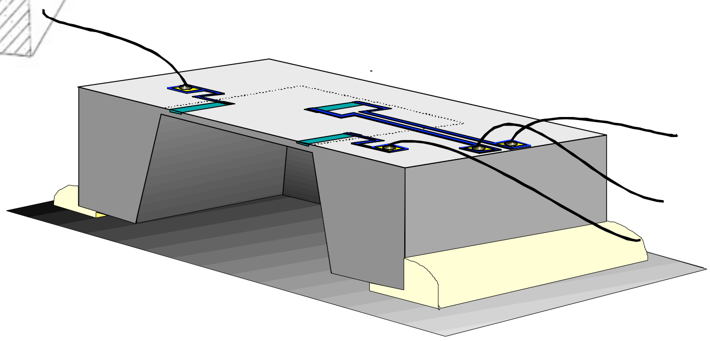




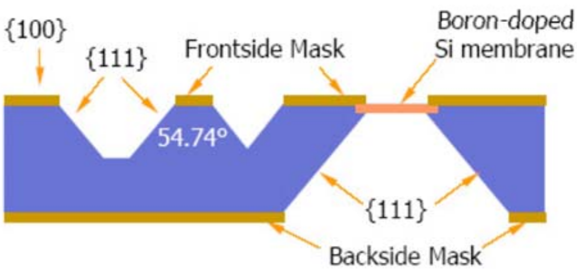
Pressure microsensors

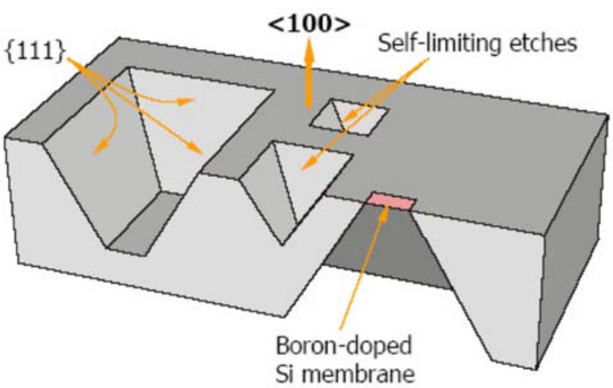


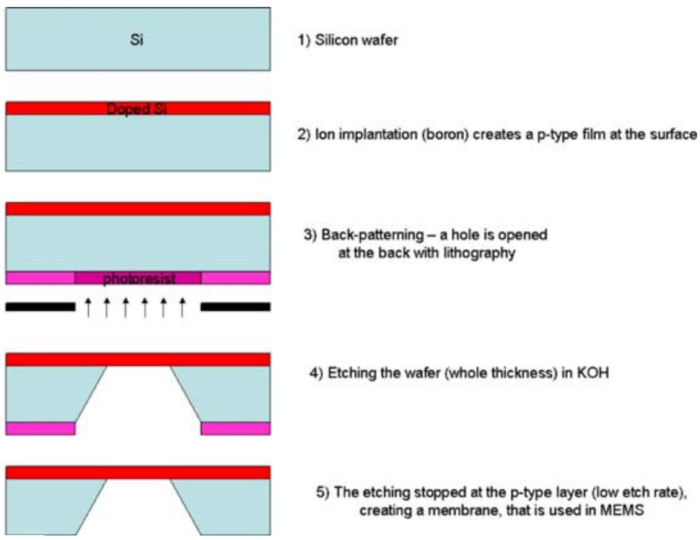




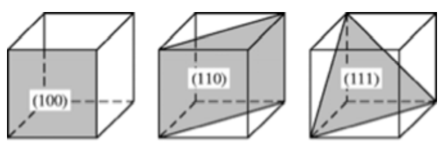
bulk micromachining



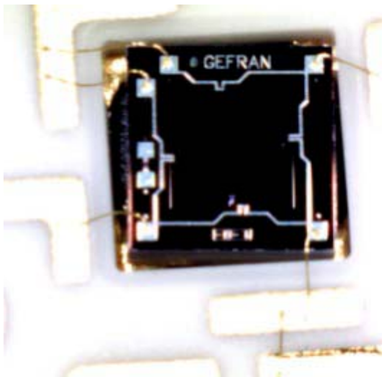




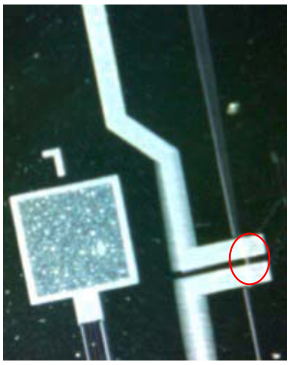
KOH selective etching $\frac{rate_{100}}{rate_{110}} = 400$



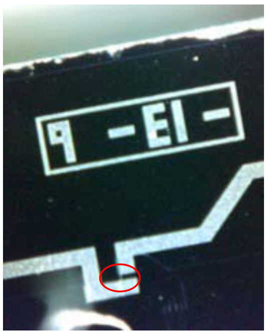
Example of pressure microsensor



orthogonal strain gauge



parallel strain gauge



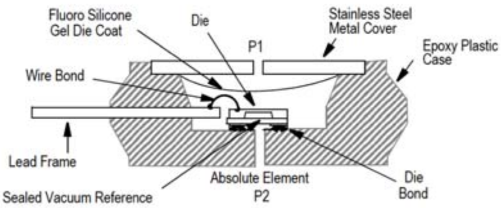
Altimeter/barometer

E=mgh

mgh$_2$ — n$_2$/V $p_2 = \frac{n_2}{V} RT;$

mgh$_1$ — n$_1$/V $p_1 = \frac{n_1}{V} RT;$

$$\frac{n_2}{V} = \frac{n_1}{V} \cdot exp\left[-\frac{Mg(h_2 - h_1)}{RT}\right]$$
$$p_2 = p_1 \cdot exp\left[-\frac{Mg(h_2 - h_1)}{RT}\right]$$
$$(h_2 - h_1) = \frac{RT}{Mg} \cdot ln\frac{p_2}{p_1}$$



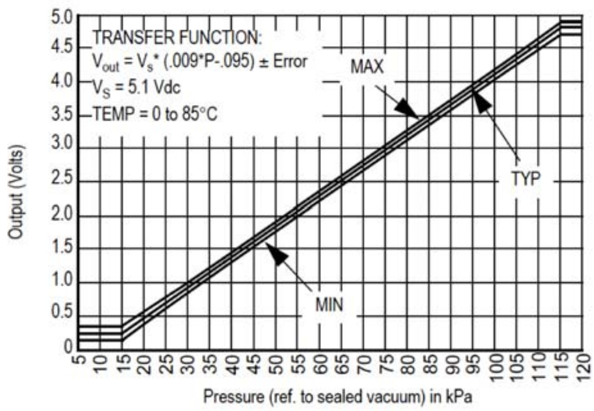
$p_{atm} = 101.325\ KPa;$
$M = 0.028\ Kg/mol$
$g = 9.8\ ms^{-2}$

$h = 1\ m$

$$p_h = p_{atm} \cdot exp\left(-\frac{Mgh}{RT}\right) = 101.313\ KPa$$

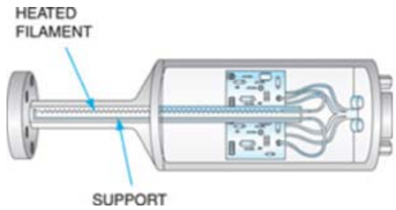
$V = 5.1 \cdot (0.009 * p - 0.095)\ V$

$V(p_{atm}) = 4.1658\ V$
$V(p_h) = 4.1663\ V$
$\Delta V = -0.5\ mV/m$



Pirani gauge

- Sensors operated at pressures lower than 1 mbar are called "vacuum sensors".
- The Pirani gauge is based on the thermal conductivity of gases. In practice the temperature of a biased wire is measured. The resistance depends on the temperature (RTD) and the temperature depends on the heat dissipation coefficient. The heat dissipation coefficient is a function of the gas pressure.
- Pirani gauges can be used for pressure in the interval 0.5-10^{-4} Torr ≅ 66-0.01 Pa



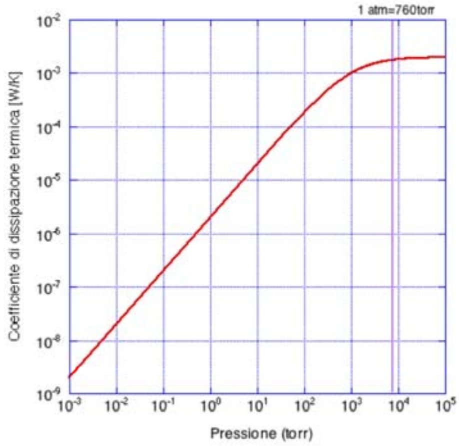
$$R = R_0 \cdot (1 + \alpha \cdot \Delta T); \quad \Delta T = \frac{V_i^2}{R \cdot \delta}$$

$$R = R_0 \cdot \left(1 + \alpha \cdot \frac{V_i^2}{R \cdot \delta}\right)$$

$$\delta = f\left(\rho_{gas}\right)$$

$$\rho_{gas} = \frac{n}{V} = \frac{p}{R \cdot T}$$

Heat dissipation coefficient vs. pressure



$$\delta = \delta_S + \delta_R + a \cdot k \cdot \frac{p \cdot p_0}{p + p_0}$$

δ_S = dissipation through thermal conduction

δ_R = dissipation through thermal radiation

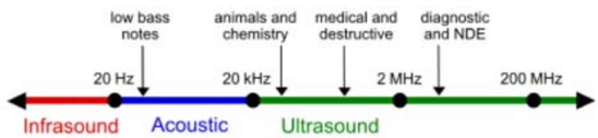
a = wire area

k = gas dependent constant

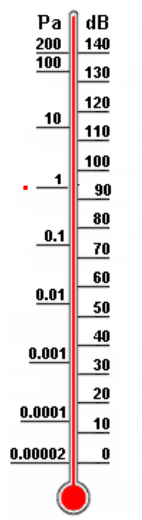
p_0 = reference pressure

Microphones

- Sound: pressure waves propagating in air, liquid and solid.
  - Human ears can detect waves between 20Hz and 20 KHz (acoustic range)
- Sound pressure
  - Difference between local average pressure and the pressure of the sound wave
  - Measured in dB
- Microphones to measure sound waves are sensors sensitive to little variation of pressure around the ambient pressure (1 atm).
- Three main categories of microphones
  - Dynamic (moving-coil)
  - Ribbon
  - condenser

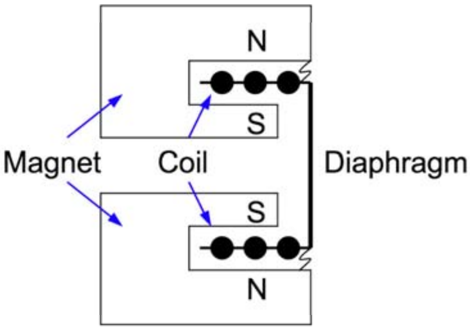


$$L_p = 20 Log_{10}\left(\frac{p}{p_{ref}}\right) dB$$

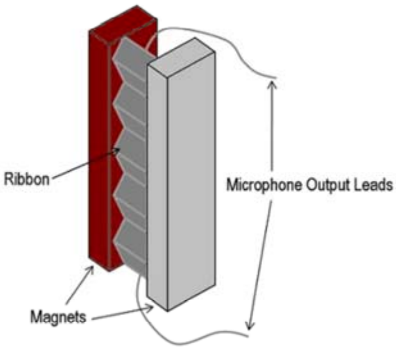


Microphones

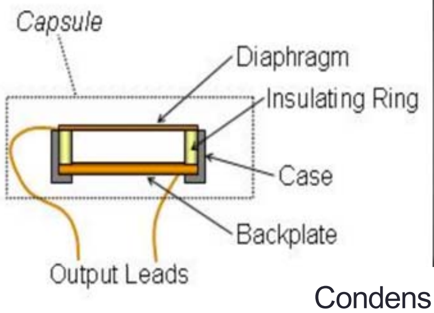
Based on electro-magnetic induction



Moving coil: peak at 5 KHz (voice)
Ribbon: flat response



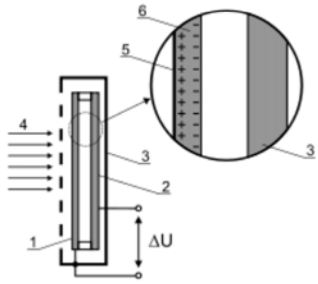
Based on capacitance



Condenser: flat response

Needs an applied voltage to generate signal

Based on electret:
It does not need voltage

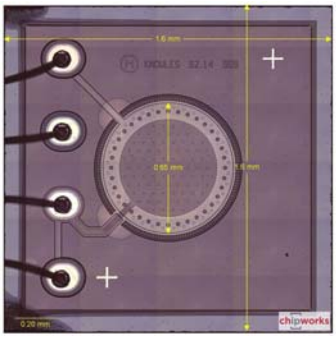


Electret: permanent electrically polarized materials (pc

MEMS microphones

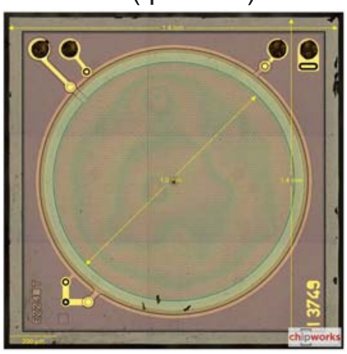
Condenser microphones for smartphones

Knowles (2006)

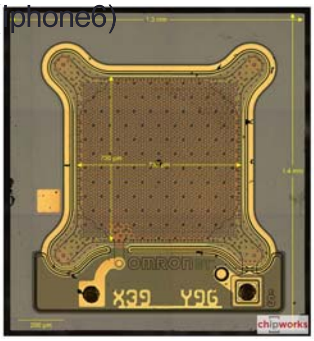


Knowles (iphone5)

Infineon (iphone6)



ST Microelectronics (iphone6)

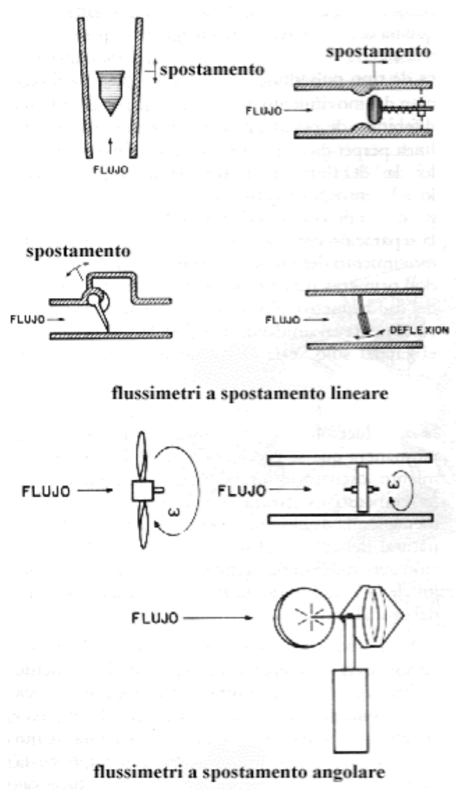


Flow sensors

- The measure and control of the flux of gases and liquids is of great importance for the control of machineries.
- Flowmeters can be based on several principles that can be applied to gases and liquids
  - Kinematic sensors
    - displacement of semi-mobile mechanical parts and deformation of flexible parts
  - Hydrodynamics sensors
    - mostly based on Bernoulli equation and Venturi effect
  - Heat dissipation

Flowmeters based on mechanical displacement

- The most simple flowmeters are based on the interaction between the flow (stream of particles) and a semi-rigid mechanical target.
- According to the constraints there are different configurations
  - linear displacement
  - angular displacement
  - angular velocity
- Sensors of position or rotation can then be applied to generate a sensitive signal.

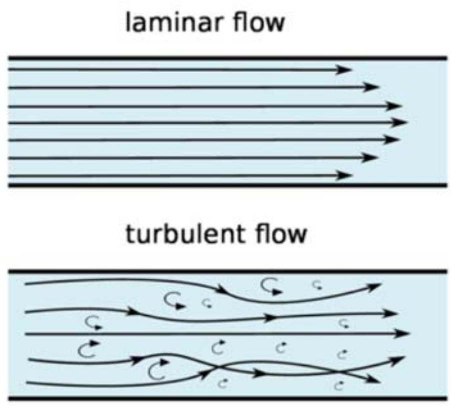


Definitions

- A flow is quantitatively expressed by the volumetric or the mass flow rate. The proportional term between them is the density of the fluid.

$$Q_M\left[\frac{Kg}{s}\right]\ Q_V\left[\frac{m^3}{s}\right];\quad Q_M=\rho\cdot Q_V$$

- The flux of a fluid forced into a pipe is not uniformely distributed. The distribution depends on the flow regime:
  - laminar flow (Poiseuille regime)
  - turbulent flow (a.k.a. hydraulic)
  - The Reynolds number (Re) defines the flow regime. It measures the ratio between intertial and viscous forces. Re depends on the flux (v), the pipe section (d), the viscosity of the fluid (μ) and its density (ρ). $Re=\frac{\rho}{\mu}\cdot v\cdot d$



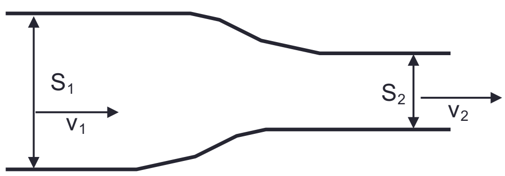
$Re<2000$ *laminar flow*

$Re>10000$ *turbulent flow*

Continuity equation

- for an insulated pipe (no addition or leakage of fluid) the quantity of mass across the section of the pipe is constant

$$v_1\cdot S_1\cdot\rho=v_2\cdot S_2\cdot\rho=Q_M$$

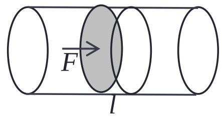


- If the density is constant (not compressible regime) the Venturi effects holds:
  - as the section decreases the velocity increases

$$v_2=v_1\cdot\frac{S_1}{S_2}$$

Bernoulli equation

la pressione è la densità di energia applicata al fluido.



$$P=\frac{F}{A}=\frac{F\cdot d}{A\cdot d}=\frac{W}{V}$$

- Energy conservation theorem applied to a moving fluid
- The total energy is conserved at any section of the pipe
  - Total energy= pressure energy + kinetic energy + potential energy
  - e= energy per mass unit

$$m\frac{P}{\rho}+m\frac{v^2}{2}+m\cdot g\cdot z=E$$

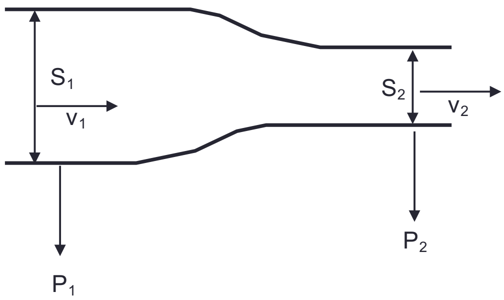
$$\frac{P}{\rho}+\frac{v^2}{2}+g\cdot z=e$$

- Application to the Venturi effect
  - The flux is proportional to the root square of the pressure drop across a constricted section of pipe.
  - case of a horizontal pipe: no potential energy variation

$$\frac{P_1}{\rho}+\frac{v_1^2}{2}=\frac{P_2}{\rho}+\frac{v_2^2}{2}\quad\Rightarrow P_1>P_2$$

$$\frac{P_1}{\rho}+\frac{v_1^2}{2}=\frac{P_2}{\rho}+\frac{1}{2}v_1^2\cdot\left(\frac{S_1}{S_2}\right)^2$$

$$v_1=\sqrt{\frac{2\cdot(P_1-P_2)}{\rho\cdot\left[\left(\frac{S_1}{S_2}\right)^2-1\right]}}\Rightarrow v_1=C\cdot\sqrt{\frac{\Delta P}{\rho}}$$

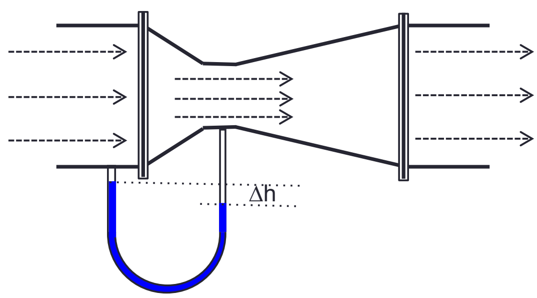


Venturi effect flowmeter

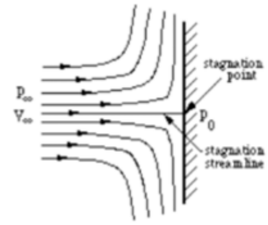
- Venturi tube
  - The fluid is forced to pass through a constricted section
  - A differential pressure sensor is applied between the unperturbed and the constricted sections
  - The flow is proportional to the square root of the pressure drop.

$$Q=C\cdot\sqrt{\frac{\Delta p}{\rho}}$$

  - C is a constant typical for the geometry of the tube, ρ is the density of the fluid.



Pitot tube



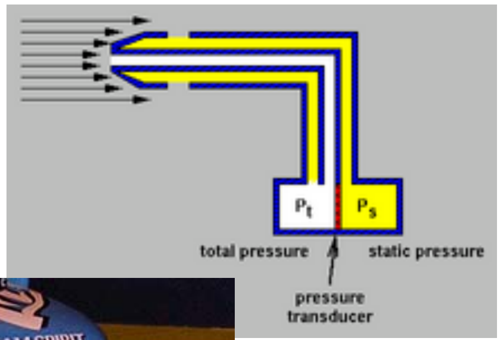
- It measures the velocity of a body supposed to be in motion in a static fluid.
- The surface of the body perpendicular to the motion is the *stagnation point*. Here the velocity of the fluid is zero and the pressure is P_0
- Applying the Bernoulli theorem between the stagnation point and the pressure of the fluid (measured in a non perturbed point) we get:

$$\frac{P_{amb}}{\rho}+\frac{v^2}{2}=\frac{P_0}{\rho} \Rightarrow P_0 = P_{amb}+\frac{\rho}{2}\cdot v^2$$

- The pressure is measured at the tip of the tube (P_0) and on the lateral surfaces (P_{amb}). the application of a differential pressure sensor between these points gets (in a non compressible fluid):

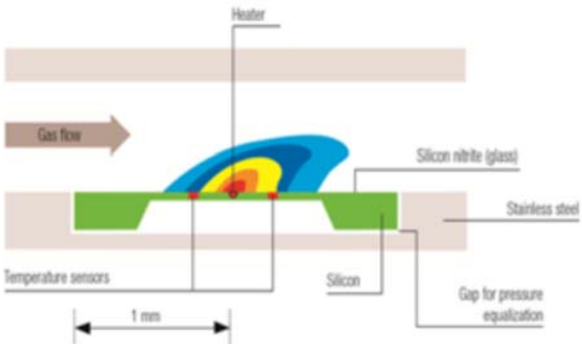
$$v=\sqrt{\frac{2\cdot(P_0-P_{amb})}{\rho}} \qquad \text{valid for } v \le 0.3\cdot v_{sound}$$

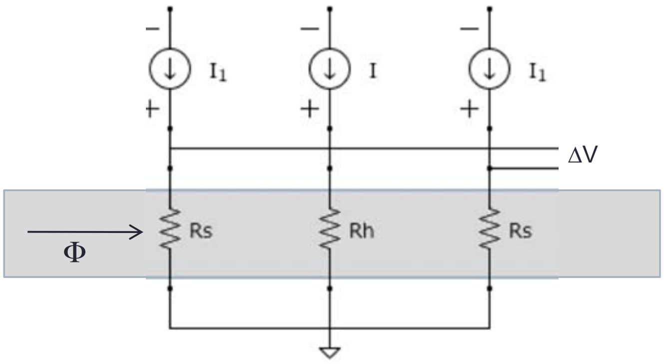
- At larger velocities the sensor can still be applied but the relationship between velocity and pressure becomes more complex.



Heat propagation flowmeter

- A flow of air (or liquid) can break the symmetry in the propagation of heat. This principle is exploited in a flowmeter using a heat source and two temperature sensors placed symmetrically respect to the source.
- The difference of temperature is a measure of fluid flux.





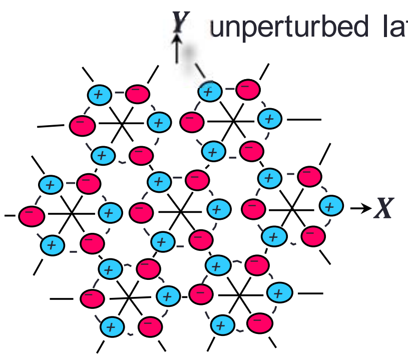
$$R_S = R_0\cdot\left(1+\alpha\cdot\left(T-T_0\right)\right)$$
$$\Delta V = \left(R_{S1}-R_{S2}\right)\cdot I = R_0\cdot I\cdot\alpha\cdot\left(T_2-T_1\right)$$

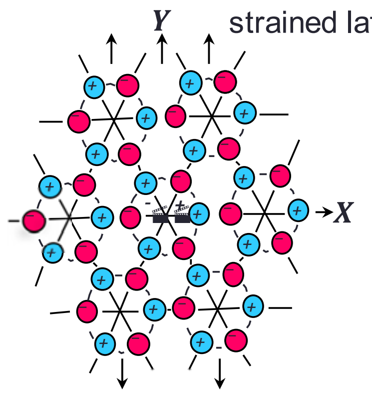
Piezoelectric sensors

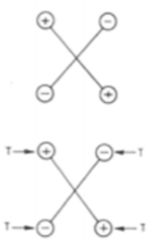
- Piezoelectric effect:
  - Curie 1880
    - the application of a mechanical stress to the surface of some natural crystals (e.g. tormaline and quartz) elicits an electric polarization proportional to the magnitude of the applied stress.
    - The phenomenon is reversible: a strain appears as a consequence of an applied voltage.
    - This class of phenomena is called Piezoelectricity from the greek word $\pi\iota\varepsilon\zeta\varepsilon\iota\nu$ which means to press.
  - For each piezoelectric material there is phase change temperature beyond which the crystal structure changes and the piezoelectricity disappears.
  - Piezoelectric material are intrinsic mechanical sensors where mechanical quantities are connected to electric quantities.

Piezoelectricity

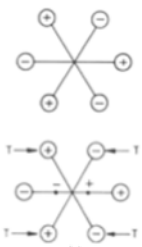
- Piezoelectricity is a property of acentric ionic crystals.
- Equilibrium position of ions corresponds to a null total electric dipole vector. The strain induced by an external stress modifies the equilibrium position of atoms and a non null electric dipole appears.



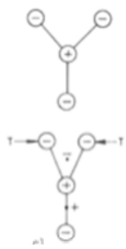




symmetric strain
no piezoelectricity

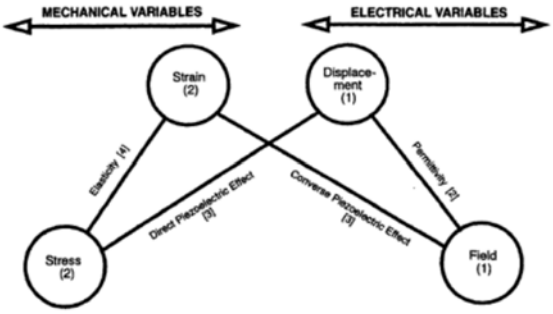


asymmetric strain
polarization parallel
to the stress



asymmetric strain
polarization
orthogonal to the
stress

Constitutive equation



- The constitutive equations of piezoelectric effect bridge the Hooke's law and the electric displacement law. The bridge quantity is the piezoelectric coefficient.
- Since in a crystal there are 6 independent directions where stress can be applied (3 principal axis and 3 rotations) the material constants are matrices.

s_{ij}: *Young's module*
ε_{lm}: *electric permittivity*
d_{ln}: *piezoelectric constant*

Hooke's law $S_i = s_{ij} \cdot T_j + d_{ik} \cdot E_k$ piezoelectric effect

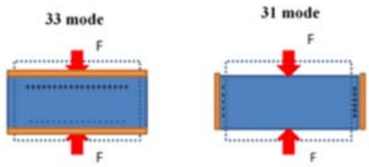
electric displacement law $D_l = \varepsilon_{lm} \cdot E_m + d_{ln} \cdot T_n$

$$j,n = 1\ldots6 \quad i,k,l,m = 1\ldots3$$

example:
Piezoelectric coefficients of PXE5 (a ceramic piezoelectric from Phillips)

$$d_{33} = 384\frac{pC}{N}; \quad d_{31} = -169\frac{pC}{N}; \quad d_{15} = 515\frac{pC}{N}$$

A torque of 1 N/m^2 along the axis 2 (component 5) elicts a charge along the axis 1 of 515 pC/m^2



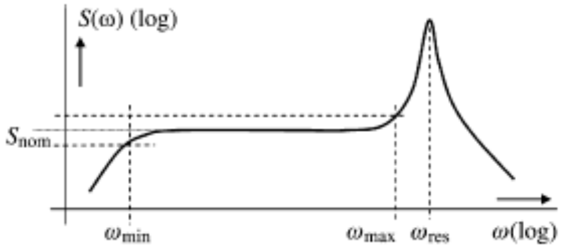
Piezoelectric materials

- Piezoelectricity is found in anisotropic crystalline dielectrics
- Synthetic materials are synthesized as ceramics (polycrystalline) structure
  - PZT: lead-zirconate-titanate ($Pb[Zr_xTi_{1-x}]O_3$ $0 \leq x \leq 1$); barium titanate ($BaTiO_3$); lead titanate ($PbTiO_3$).
  - the global piezoelectricity appears after the application of poling (exposure to a large electric field).
  - piezoelectric polymers, e.g. polyvinylide fluoride (PVDF)

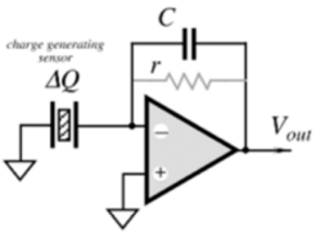
material	density (kg/m^3)	Curie temperature (°C)	ε_{11}	ε_{33}	piezoelectric coefficient (pC/N)	resistivity (Ωcm)
quartz	2649	550	4.52	4.68	d_{11}=2.31 d_{14}=0.73	10^{14}
PZT	7600	285	1730	1700	d_{33}=425	10^{13}
PVDF	1780	100-150		10-13	D_{31}=-18	10^{15}

Application of piezoelectric materials

- Piezoelectric materials are used to actuate and to sense forces, deformations, pressure and acceleration.
- In the typical measurement configuration the generated charge is converted in voltage by a charge amplifier
  - amplifier with a very large input impedance
- The frequency response has a resonance peak at high frequency (due to the stiffness of the material) and then a large low-pass band.
- The d.c. response is null due to the finiteness of the load resistance.

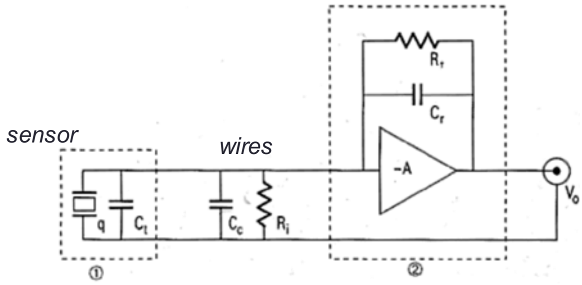


Measurement principle



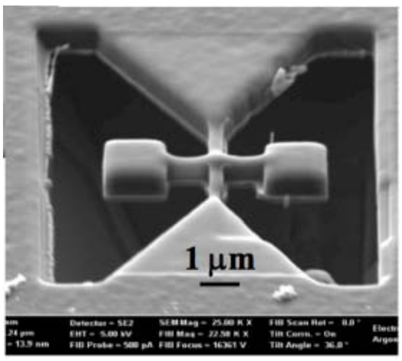
$$\frac{dQ}{dt} = -C \cdot \frac{dV_0}{dt} \Rightarrow \Delta V_0 = \frac{\Delta Q}{C}$$

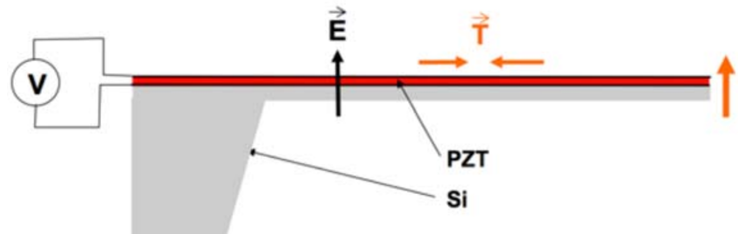
$$C = 1\,pF \Rightarrow S = 1\frac{V}{pC}$$

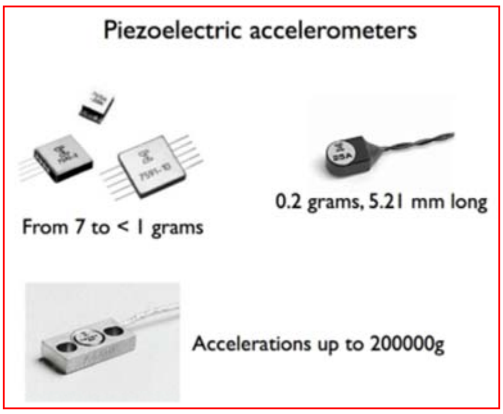


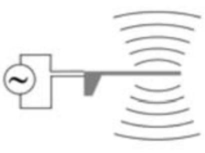
Integrated piezoelectric sensors

- PZT is compatible with the silicon technology, then it can be used for silicon based micromechanical sensors.

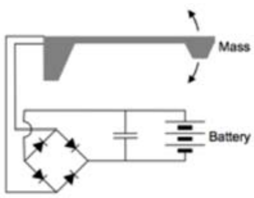




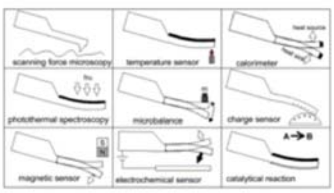




Actuator and sound emitter

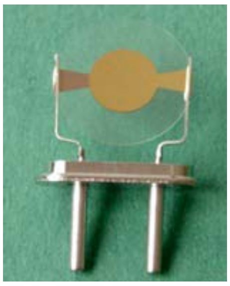


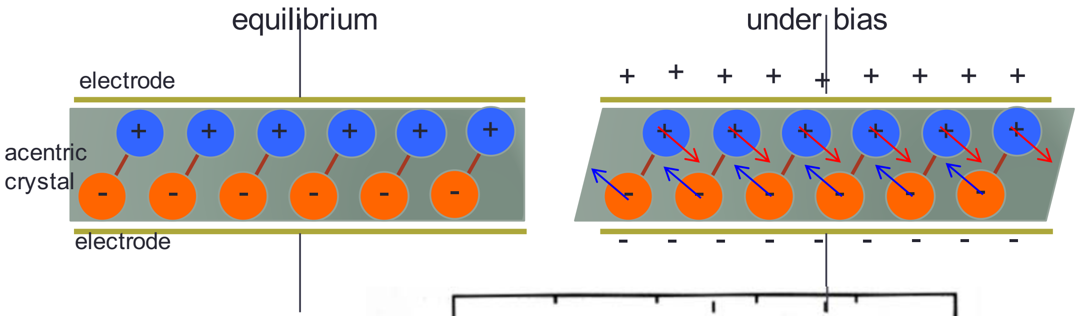
Energy harvester

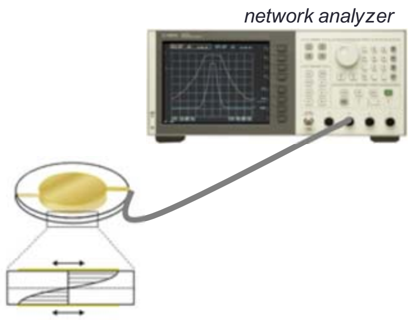


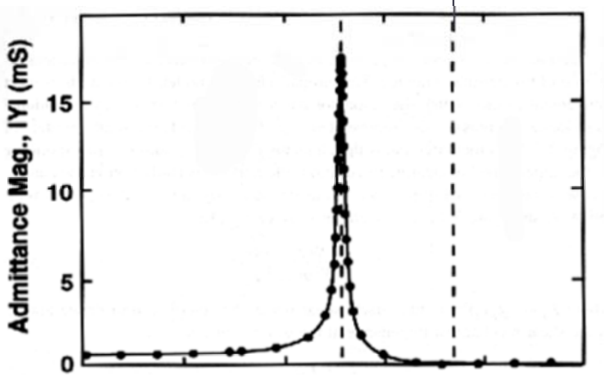
Sensor (Grabiec, 2005)

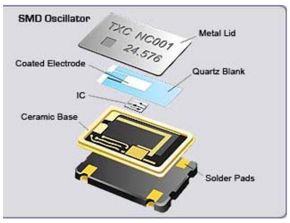
Thickness shear mode resonator











TXC

Model	Frequency	Stability (-10~70ºC)	Operating Temp
7CZ	32.768KHz	±25ppm	-40~+85ºC